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A paper to be presented at the 150th Meeting of the American Institute of Electrical Engineers, New York, January 25th, 1900.

[ADVANCE COPY—SUBJECT TO REVISION.]

A NEW PAGE-PRINTING TELEGRAPH.

BY WILLIAM B. VANSIZE.

The problem of automatically recording telegraph messages in Roman type is one that has fascinated inventors almost from the days of Morse. Royal E. House invented a type printing telegraph, which was in successful operation in this country in competition with the Morse and Bain systems prior to 1857. Many years ago the stock-ticker was perfected, and the Hughes, a tape printing telegraph, and several modifications of it have been employed for a long time with success by various telegraph administrations, chiefly on the Continent of Europe. But it is only within the last year or two, almost at the close of the 19th century, that the problem of constructing a long-distance, high-speed, page-printing telegraph has been solved. A printing telegraph of this kind has recently come to my attention.

It has received the commendation of practical men in the telegraph service, and is believed to be of such simplicity and merit as to win for itself and maintain a position among the apparatus and instruments in constant use on commercial telegraph lines. The arrangement referred to has been devised and constructed by and under the direction of Donald Murray, an Australian journalist, without prior practical experience in telegraph work.

A Retrospect.

Looking back over the last thirty years, a substantial part of which time has been spent in the service of the large telegraph corporations, I can recall but comparatively few arrangements of apparatus which have passed the experimental stage and gone into commercial use. In 1869 there was the well known Morse

key and spaced alphabet, with the relay and sounder, and in some few instances the embossing register at way stations. In 1872 or 1873, circuits were equipped with balanced receiving apparatus and continuity preserving transmitters for duplex transmission. A little later, reversals of current were used for one set of signals and variations in current strength for another set of signals, affording two transmissions in the same direction. By duplexing this last named arrangement the quadruplex was brought to a more or less satisfactory condition through the efforts of Edison, Nicholson, Prescott, Gerrit Smith and others. About 1875 there was introduced on the lines of the Atlantic and Pacific Telegraph Company an automatic system of transmission improved by Little and Edison, employing a perforated paper strip at the transmitting station, and a chemically-prepared paper strip to record the Morse characters at the receiving station. Some years later the Wheatstone system was first employed by the Western Union Telegraph Company. Later Mr. Delany's arrangement for distributing a single main line between a number of sending and receiving operators at separated stations was employed on a special line between Boston and Providence. Numerous systems of printing telegraph, recording upon a strip of paper and upon a page have been patented and exploited by corporations organized for that purpose. The Phelps motor printer, in this country, and the Hughes printing telegraph upon the Continent of Europe, both recording upon a strip, have had more or less extensive employment.

It is undoubtedly objectionable to use a perforating strip for transmitting purposes; but when to this is added a received record to be translated, there exist objections which only other merits, as in the case of the Wheatstone apparatus, partially compensate. At the other extreme is the simple Morse requiring no added time interval for translation at either end of the circuit; the operation of transmitting and recording being substantially co-extensive with the time in transit.

Recent Developments.

The later improvements in printing telegraphs have been directed to finding a substitute for the direct manual operation of the key-board and the employment of automatic transmission, like that of the Wheatstone system, by the use of perforated tape. The work of the operators in transmitting messages is

thus first stored in the perforated tape, produced at any convenient speed, and the automatic transmission may take place at a high and uniform rate, the line and receiving instruments being worked to their full capacity. Another means for overcoming the delay due to direct manual transmission is to divide up the use of the line among several operators, a locking device holding the line for an interval at the service of each in turn. In this case while the speed of the operator may be slow, the total use of the line and instruments is greater than one operator could achieve working at his highest rate. There are two arrangements of apparatus now receiving attention employing the transmitting tape; that of Charles L. Buckingham, and that of Donald Murray, the latter here under consideration. The French system of Emile Baudot and the American system of Professor Rowland employ an arrangement for dividing the use of the line among several operators.

What telegraph officials really need is the simplicity of the Morse system combined with increased speed of transmission and economy of time between the transmitting customer and his addressed correspondent. Up to the present time nothing has surpassed the Morse for this purpose. We may economize time on the wire by the Wheatstone, by the chemical automatic system referred to, or by the use of Delany's multiplex system; we can provide for a special class of telegraph business by adopting some of the high speed transmitting systems like those contemplated by Professor Crehore, or by Delany's modification of the chemical automatic. For these, high speed systems requiring preparation for transmission at one end and translation for delivery at the other end, a demand must be created by the requirements of a class of business to be transmitted and delivered faster than the mail, and somewhat slower than the transit time of the Morse system.

In the recent development of Morse there is an analogy between the contest between armor and armor-piercing projectiles on the one hand and the transmitter and receiver of the Morse code on the other. In the case of armored vessels an increase in the thickness or resistance of the armor has been answered by increased capacity for penetration by the projectile, and vice versa. In the early days the embossing or recording register could easily record the most rapid Morse sender. When receiving by sound it is an effort to keep up with the

speed of a rapid sender, where the receiver uses pen to record in script. When the typewriter is substituted for the pen, the receiver can more than keep pace with the sender; if we introduce automatic Morse transmission we surpass the power of the human ear to receive and translate to the typewriter. If we transmit mechanically and record mechanically by means of a typewriter we shall have eliminated these human agencies with their limitations, and that is what Mr. Murray has done in a very complete and satisfactory manner.

Before further considering Murray's arrangement I want to assign due credit to my friend Chas. L. Buckingham, who has a marvelously ingenious system in commercial use on the lines of the Western Union Telegraph Company. The received record is made in print on the ordinary telegraph blank; the transmission is by means of a perforated paper strip. It has a maximum capacity of about 100 words a minute on a circuit between New York and Chicago. The circuit can be duplexed to double the stated capacity. Its use, I believe, is successful, and commercially so.

Murray's System—Transmitting Arrangement.

Mr. Murray employs a special alphabet, perforating the transmitting tape with a key-board perforator, having a separate movable lever for each character; each character occupies an unvarying linear space on the tape, and consists of five perforated and unperforated subdivisions of such space. The difference in the number and succession of these subdivisions or perforations imparts the designating characteristics. There are no spaces between successive letters or characters. Either makes and breaks, or reversals can be used in transmitting. It is to this fundamental fact—all letters of the same length—that the success of the system is due. Each letter occupies half an inch on the transmitting tape, and a similar length on the receiving tape. The result is that a comparatively simple transmitting tape perforator worked by an ordinary typewriter key-board is rendered possible; there is no necessity for employing a complicated differential paper feed mechanism to provide for letters of varying length. In connection with the ordinary typewriter key-board there is a group of ten punches, one punching magnet, and one spacing magnet, controlling a motor-driven escapement.

The Receiving Apparatus.

At the receiving station there is an electro-magnetic perforating device which accurately reproduces the transmitting tape by producing corresponding perforations and spaces. This received perforated tape passes from the receiving perforator into the typewriter operating device. This typewriter operating device consists of five longitudinally reciprocating bars or "combs," 39, Fig. 1, presenting five pointed terminals, 40, to a perforated plate or die, 38. The perforated tape passes between the surface of the perforated plate and the pointed terminals of the bars. The pointed terminals of these bars register respectively with the five holes in the die. The tape is moved along between the die and the pointed ends of the bars step by step, the length of a letter or character at each step, say one-half inch. When perforations in the tape coincide with the pointed ends of the bars and corresponding perforations in the plate and die, and the plate is moved toward the pointed ends of the bars, the bars are or may be separated into two groups; one group is moved longitudinally, corresponding with the unperforated subdivisions of the tape, the other group project through the perforations in the tape and in the die, and are unmoved. Lying over the five bars or "combs" at right angles thereto, are a series of thin metal strips, 41; each strip is mechanically connected with its individual key lever on the typewriter. The upper surface of the five bars first described are notched arbitrarily. These notches are caused to be aligned below any one of the strips under the control of the perforated tape and die; when any one of the strips drops into a groove a motor-driven cam engages it and produces a movement of the typewriter lever. The movement of the die and paper tape and of the typewriter key-lever is produced by motor-driven cams. It will be seen that this mechanism will operate not only a typewriter, but any keyboard machine such as a typesetting machine or linotype. The perforated receiving tape is therefore available for setting type automatically.

The General Diagram.

In the general diagram (Fig. 1) shown there is the apparatus at the transmitting station connected by a single main line 9 with the apparatus at the receiving station. I have placed a double marginal line around the apparatus at the transmitting station to distinguish it. The vibrating reed 1 is in a local cir-

cuit with an electromagnetic motor 2. The reed makes and breaks its own circuit, and is substantially like the well known LaCour

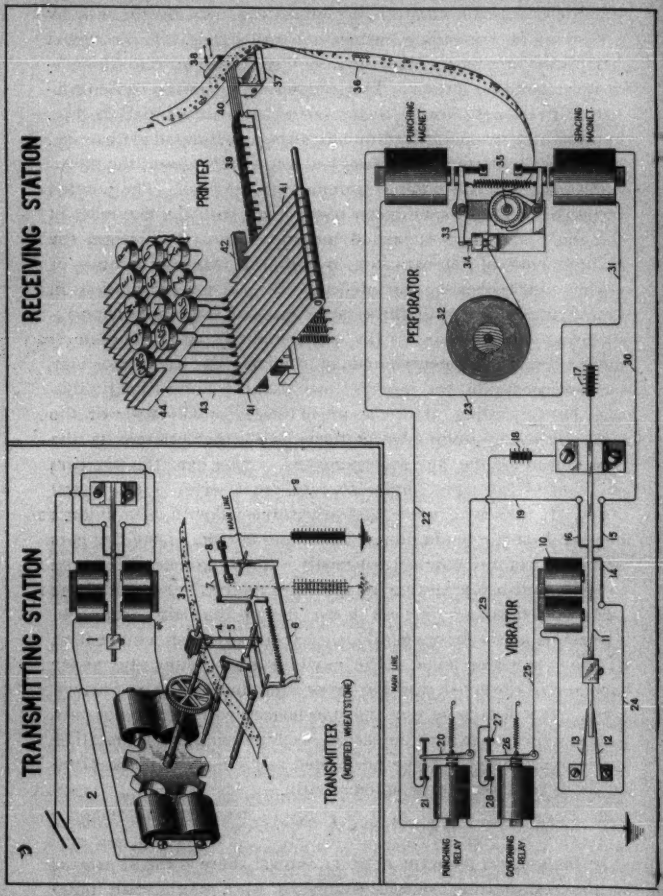

FIG. 1.

phonic wheel device. The "prickers" 4 and 5, familiar features in the Wheatstone transmitter are located as usual in line with the advancing lines of perforations in the transmitting tape 3;

6 and 7 are reciprocating rods engaging respectively with opposite terminals of the centrally pivoted pole changing switch arm 8. The parts shown are all essential parts of the well known Wheatstone transmitter, which is here used practically without alteration, except that the prickers 4 and 5 are arranged to move or reciprocate together instead of alternately, thereby enabling any described multiple of a single impulse to be transmitted. The ordinary arrangement of the Wheatstone transmitter can only transmit unit signals or odd number multiples thereof; it is practically an alternating current generator. The Wheatstone signals have to be built up by omitting a positive or negative impulse, hence an ordinary Wheatstone transmitter can transmit a dot or a dash equal to three or five dots, but cannot transmit a dash equal to two or four dots. Mr. Murray, to avoid this difficulty, arranges the prickers to reciprocate together instead of alternately. There is thus obtainable transmitted impulses or dashes equal to one, two, three, four, or five dots, with corresponding spaces.

In transmitting, the number of impulses thrown upon the main line is minimized by producing the impulses locally at the receiving station, and employing only sufficient main line impulses to determine the action of the perforator. At the receiving station there is, therefore, a main line relay to determine the action of the punching magnet, and a governing relay which operates to maintain unison between the main line impulses as they arrive and corresponding impulses in the local circuit. For the purpose of creating these local uniform impulses there is a vibrating reed, 11, operated by an electro-magnet, 10. The circuit of this magnet extends from the local battery 18 through the reed 11, contact point 14, wire 24, magnet 10, wire 25, armature 26 of the governing relay, thence by way of the points 27 or 28, and wire 29, to the battery. The precise operation of the governing relay I will describe presently. The receiving perforator is composed of a punching magnet and a spacing magnet; the punching magnet operates a spring-retracted pivoted armature bar, 33, mechanically connected with the punch 34, reciprocating through a guide block and engaging the tape 32 upon the surface of a suitable die, over which the tape passes. The tape is fed along by a star wheel located on a motor-driven shaft, 35, and upon this shaft is an anchor escapement under control of the spacing magnet. The vibrating

reed 11 alternately makes and breaks the local circuit of the spacing magnet; this circuit extends from battery 17 to the reed 11, contact spring 15, wire 30, through the magnet and wire 31 to the battery. The punching magnet is in a local circuit with a break point operated by the vibrating reed 11 and controlled by break points of the punching relay, so that while the reed is continually generating local circuit impulses, these impulses are effective to operate the punching magnet at such times only as the punching relay is closed upon its front contact. This local circuit passes from the battery 17 through reed 11 to contact point 16, thence by wire 19, armature bar 20, contact 21, wire 22, through the punching magnet, and wire 23 to the battery. It will thus be seen that the reed 11 is continually making and breaking two circuits alternately; first, that of the spacing magnet, which is a continuous operation; and second, that of the punching magnet, which is an intermittent operation, rendered so by the action of the punching relay. This punching relay and also the governing relay may be either neutral relays responsive to makes and breaks, or they may be polarized relays responsive to reversals of current. As the reed 11 vibrates, the electric impulses in the spacing magnet circuit permit a steady progressive movement of the tape 36. Upon the arrival of an impulse of current from the transmitting station, the contact points 20 and 21 of the punching relay are held closed for one, two, three, four or five times the time interval of one dot length, and while this relay circuit breaker is closed, the punch 34 operates to perforate the tape as many times successively as permitted by the time length or duration of the transmitted impulse upon the main line. Mr. Murray has thus avoided the necessity of transmitting over the main line all impulses necessary to produce spacing, and all but a fractional part of the impulses necessary to produce the perforations. It is of vital importance, however, to preserve unison between the arriving transmitted impulses in the main line and the local punching and spacing impulses at the receiving station. This is done in the following manner:

How Unison is Maintained.

The governing relay operates a circuit breaker, 26, moving between two fixed contacts, 27 and 28, electrically connected to the same circuit terminal, so that the moving contact in going

from one to the other operates to open the circuit during its time of transit only. I have referred to this briefly as "armature transit time." This break in the local vibrator circuit takes place at the beginning and end of each main line signal, and as the main line signals arrive at a uniform rate and are of unit or multiple unit duration, the governing relay operates its break-point at uniform unit intervals or multiples of these intervals. In the same circuit in which this break-point operates, there is also the break-point 14 of the motor magnet 10, which works on the familiar buzzer principle. There are thus two break-points in the same circuit. If they open and close together, then full vibratory impulses flow through the motor magnet. If, on the other hand, the rate of vibration of the reed tends to accelerate, or the rate of the arriving current signals tends to lag, then the two breaks occur more or less alternately and consequently less current gets through—the impulses are clipped—and the rate of vibration of the reed is reduced. In practice, the receiving vibrator is set to go one or two per cent. faster than the rate of the arriving signals, and then the governing action of the two interfering break-points in the same circuit results in the establishment of a steady dynamic balance between the accelerating tendency of the reed and the retarding tendency of the arriving main line signals. By this arrangement the necessity for sending correcting impulses over the main line to secure synchronism is avoided, the correcting impulses being obtained locally with the co-operation of the main line signals themselves.

It is to be understood that movable weights are present upon each reed, that of the transmitting station and that of the receiving station, and by varying the position of the weight upon the reed, the rate of vibration and the rate of transmission may be changed. At the receiving station there is provided means whereby the rate of vibration of the reed may be varied in harmony with variations in current in its motor magnet circuit. Under normal conditions with a motor magnet and circuit operated by break points carried by the reed, an increase of current, resulting in increased magnetic effort, will decrease the rate of vibration, and, conversely, a decrease of current resulting in decreased magnetic effort will accelerate the rate of vibration. This is believed to be due to eddy-currents generated by the magnetic field of force and the movement of the reed in such field. The action described while it has a damping effect, is of

no practical use as a means of regulation for the work to which the reed is here put, because of its exceedingly limited range. It is necessary in maintaining unison to have a considerable range of variation in the speed of this reed, such variation in speed to be attained in response to variation in the length of current impulses of uniform strength.

Constrained Vibration of the Reed.

To secure this result there is placed at or near the free end and upon the opposite sides of the reed, resilient stops, shown at 12 and 13. These springs receive the reed on each side with a cushioning effect and impart an initial return movement. In explanation of this result it is to be recalled that the rate of vibration of a reed varies with its length, mass, and the distribution of such mass; increase of current in the motor magnet circuit increases the amplitude of vibration without varying the speed beyond the slight electro-magnetic damping effect already described. The energy imparted to the reed by the motor magnet is dissipated by the air resistance and molecular friction in the reed; an increase in the propelling energy brings a corresponding increase in the dissipation noted. Therefore, while the amplitude of vibration is widened, the speed is not increased. If fixed limiting stops be provided, to limit the amplitude of vibration, the energy of the reed cannot then be dissipated in the manner first described to the same extent, and there results an increase in the rate of vibration, while a portion of the energy imparted to the reed is dissipated in heat generated at the point of impact. To avoid this loss and further increase the rate of vibration, the energy which would otherwise be lost at a rigid contact is stored in the resilient devices, 12 and 13, already referred to, to cushion the impact of the reed, and impart an initial return movement. By this expedient, the movement of the reed is rendered smooth and uniform; it is freed from the interference due to the impact with a rigid stop, which acts to jar and disturb the normal rate of vibration, and its rate may be varied by varying the current impulses.

In further explanation of this device we may recall the fact that, within the limits of elasticity, a helical spring varies in length in arithmetical ratio with the force extending it; that is to say, if a helical spring extends one inch with a weight of one pound, it will extend two inches with the weight of two pounds,

and so on. Within restricted limits the same rule holds good with simple flat springs of uniform thickness, like those commonly employed for electrical contact purposes. It has been found by experiment that springs of this kind, whether arranged for tension or for compression, do not vary the rate of vibration of the motor-driven reed to any appreciable extent in response to variations of current in the motor magnet circuit; but if resilient stops are constructed and arranged, like 12 and 13, to rapidly compound their resistance to the impact of the reed, they are practically operative for the purpose of varying the rate of vibration in response to variation in current.

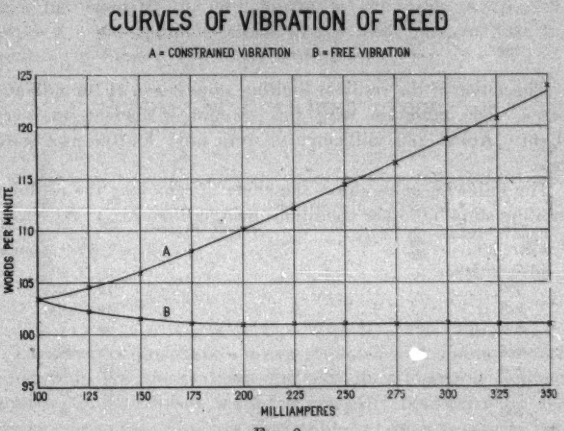

FIG. 2.

CURVES OF VIBRATION OF REED.

I have here a diagram [Fig. 2] showing the action of a reed when free to vibrate without limiting stops, as compared with the action of the same reed operating under the constraint imparted by the tapering springs, called resilient limiting stops. The figures were obtained by varying the amount of current flowing through the reed magnet coils and then measuring the amount of tape fed through the perforator under control of the reed. The tape is fed forward one step for each vibration of the reed, and its extent therefore gives an exact record of the rate of vibration per minute. The receiving vibrator and other

instruments were set to transmit and record at the rate of 112 words per minute, and the current in the reed magnet was then varied with the following result:

Current in Milliamperes.	Tape fed per minute. ft.	in.
350	24	0
325	23	4⅔
300	23	0
275	22	6¾
250	22	1¾
225	21	8½
200	21	4
175	20	11
150	20	6
125	20	3
100	20	⅓

The action of the resilient limiting stops began at 100 milliamperes. The vibration with this amount of current was very slight. About 230 milliamperes were used for the 112 word rate.

The following table shows the effect of removing the resilient limiting stops, all other conditions being as before.

Current in Milliamperes.	Tape fed per minute. ft.	in.
350	19	6½
325	19	6½
300	19	7
275	19	6½
250	19	6¾
225	19	6½
200	19	6
175	19	7
150	19	8
125	19	9½
100	20	⅓

The reed did not vibrate reliably under 100 milliamperes, and even with that current it was feeble. The vibration did not become fairly strong until 125 milliamperes was reached. The figures and the curves indicate that the speed increases with decrease of current below 150 milliamperes. This was possibly due to the electro-magnetic damping effect of the iron armature vibrating in the magnetic field. Actual trial has shown that this effect is too feeble and unstable to be used for governing the rate of the reed by possible variation in the current. A more posi-

tive and reliable result was required, and it was supplied by the resilient limiting stops. It will also be seen by the diagram that above 150 milliamperes the rate of vibration of the reed is uniform irrespective of the amount of current when the reed is not constrained by the resilient limiting stops. This is indicated in the straight horizontal portion of the curve B. Curve A shows the effect of the resilient limiting stops in causing an almost uniform acceleration of the speed of the reed in accordance with the increase of current. With respect to the curve A the variation in words per minute between extreme points is from 103 words with 100 milliamperes to 123 words with 350 milliamperes, and with respect to the curve B the variation in words per minute is from 103 words at 100 milliamperes to 101 words at 350 milliamperes, a perceptible decrease in the word rate, with an increase of $3\frac{1}{2}$ times in the strength of current.

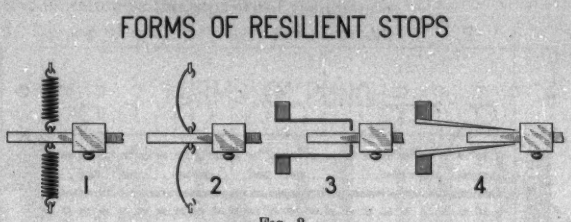

FIG. 3.

The forms of limiting stops experimented with and heretofore referred to are shown upon the diagram "Forms of resilient stops" [Fig. 3]. Nos. 1 and 3 are practially ineffective for the purpose intended; No. 2 is susceptible of some beneficial use, while No. 4 is a practically successful arrangement, and the one to be preferred.

THE UNISON DIAGRAM.

I have here a "Unison Diagram" [Fig. 4] to illustrate the relative position of the current impulses in securing unison. There is shown a section of receiving-tape in which appears the word "Paris," the division between the letters being marked with a dotted line. Below the line of the tape is shown the corresponding conditions of current in the local and main line circuits; when in unison, and again when out of unison. In line 1 there is shown the current impulses in the punching magnet

circuit effective to produce the perforations shown on the section of tape; line 2 shows the current impulses in the local spacing current; at line 3 is shown the main line impulses due to the operation of the automatic trasnmitter effective to produce the perforation in the tape represented; at 4 is shown the operation of the armature of the governing relay, the unshaded divisions representing the "armature transit time," and the relative position of the armature transit with respect to the impulses in the main and local circuits; at line 5 is shown the impulses in the motor magnet circuit which operate to drive the reed; these impulses are shown as in unison with the main line and local circuit impulses. In line 6 is shown the same motor magnet circuit impulses out of step, or out of unison. The continuous operation

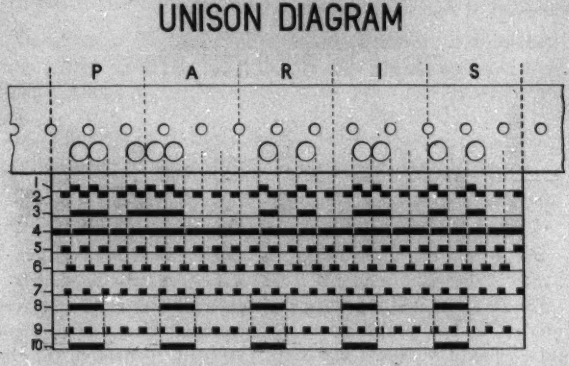

FIG. 4.

of the unison device is best illustrated in the actual record taken upon a chemically prepared paper tape with a pair of iron pens, one in the local circuit and one in the main circuit; the pens were arranged side by side; line 7 shows the record of impulses in the motor magnet circuit; line 8 shows the main line impulses, the action of the armature of the governing relay having been suppressed. In lines 9 and 10 the same means are employed to render graphic the impulses in the two circuits, and the operation of the transit of the armature bar 26 of the governing relay is also apparent in clipping the local circuit impulses, and thus curbing their motive power effect on the reed. There

is thus established a continuous "dynamic balance" between the tendency of the reed to accelerate its speed and the tendency of the main line signals to retard its speed. The shape of the punch for perforating the receiving tape is quite important. A large number of forms were tried. The diagram "Forms of Punch" [Fig. 5], shows some of the leading shapes. No. 5 is much the best.

Telegraphic Alphabets.

In considering the merits of this arrangement of apparatus and in accounting for the want of commercial success of so many other arrangements, it may be said that printing telegraphs have not been generally employed for the following reasons, among others:

1. Delicacy and complexity.
2. Slowness of operation.
3. Limitation in the capacity of the operator.
4. The use of a tape for the received record.

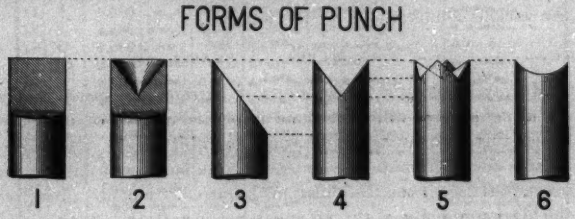

Fig. 5.

Recently some arrangements of apparatus have been devised which overcome these difficulties by two radically different methods; *i. e.*, automatic transmission and multiple transmission. The first uses the Wheatstone transmitter and perforated tape; the second employs direct manual transmission distributing the time for using the line between a number of operators. The slowness of operation has been due not only to the limitations of the operator, but also to the character and succession of the line impulses employed. In long distance working and with the growing use of underground and ærial cables, this is of extreme importance. The reliable material for high speed signaling over long distances in printing telegraphy consists prac-

tically of five or six variations in the time interval; two variations in direction, and one variation in strength, *i. e.*, makes and breaks. Two very desirable conditions are that letters shall be of uniform length, and that either makes and breaks or reversals shall be used, but not combinations of the two. The Morse alphabet is ideal except for the great variation in the length of the characters.

The importance of an alphabet having letters of equal length has been generally acknowledged, and many designers have adopted the only practical arrangement, namely, thirty-two possible combinations of five positive and five negative elements taken five at a time. Such an alphabet was first suggested by Gauss and Weber of Gottingen in 1833. It was revived in 1874, when Baudot and others employed it for their multiplex printing telegraph systems. Baudot's arrangement, which may be taken as typical of this class of equal letter alphabets, is shown in the diagram called "Table of Alphabets" [Fig. 6]. It has the disadvantage of using makes and breaks as well as reversals, necessitated by the use of equal units of current to form the various combinations for the letters.

The Murray Alphabet.

Murray by using multiple units of current and space (that is by using several different time intervals instead of one only) avoids this disadvantage and has not found it necessary to use reversals, five different time intervals combined with makes and breaks giving all the combinations required. He has preserved the ideal simplicity of the Morse alphabet with the added advantage of securing letters of the same length. The Murray alphabet appears in the table; it will be seen that the uniform time for each letter is divided into five equal units or subdivisions, one or more of these five subdivisions being a current impulse, so that we get current impulses or spaces of one, two, three, four or five units duration. Thirty-two possible combinations are obtained in this manner, and by using two of these letter signals as prefixes to the others, capitals, figures, and lower case letters, about eighty-seven characters, may be transmitted. Makes and breaks or reversals may be used, therefore, adapting the system for use in quadruplex transmission; a use not practicable with an alphabet employing both makes and breaks and reversals. The alphabet is however only available for machine telegraphy,

as it is practically impossible to observe five different time intervals with sufficient accuracy in manual transmission. No space is required between letters in the Murray alphabet, whereas in the Morse a three unit space follows each letter. This is shown in the "Table of Alphabets." It will be seen upon examination that the maximum number of impulses required is

TABLE OF ALPHABETS

	LETTERS	FREQUENCY	"MURRAY" SIGNALS	"MURRAY" TAPE	"BAUDOT" ALPHABET	"INTERNATIONAL" MORSE	"AMERICAN" MORSE
1	e	14,000		o····			
2	t	10,000		····o			
3	a	9,000		oo···			
4	i	9,000		·oo··			
5	n	8,000		··oo·			
6	o	8,000		···oo			
7	s	8,000		o·o··			
8	r	7,000		·o·o·			
9	h	6,000		···o·o			
10	d	5,000		o··o·			
11	l	5,000		·o··o			
12	u	4,500		oo·o·			
13	c	4,000		·ooo·			
14	m	3,000		··ooo			
15	f	3,000		o·oo·			
16	w	2,500		oo··o			
17	y	2,500		o·o·o			
18	p	2,400		·oo·o			
19	b	2,000		o··oo			
20	g	2,000		·o·oo			
21	v	1,500		·oooo			
22	k	800		oooo·			
23	q	600		ooo·o			
24	j	500		oo·oo			
25	x	500		o·ooo			
26	z	300		ooooo			
27	,	4,500		o···o			
28	.	3,000		oo·o·			
29	Space Key			···o·			
30	Capital Key			·o···			
31	Figure Key			···o·			
32	Release Key			·····			

BAUDOT
1 = E
2 = "ERROR"
3 = NOT USED

FIG. 6.

three for the letter y, and the average number reckoned according to the frequency of the letters is 1.25 impulses as against 2.59 for International Morse. As compared with this 1.25 impulses per letter, a system employing the Baudot alphabet has to transmit not less than five, and this does not take into account

the necessary correcting impulses to secure or maintain synchronism. Murray has the shortest alphabet possible to be constructed from reliable signaling material and by combining it with machine transmission the entire time of the line is used in the most advantageous manner possible.

Comparison of Alphabets.

Referring to Fig. 7, "Comparison of Alphabets," it will be seen that the Murray word is shorter than the Morse in the ratio of 30 to 51, or 41.18 per cent. This theoretical diagram is borne out by a comparison made with an actual word such as PARIS. This comparison is made on lines 13 and 14. It will be observed that in this case the Murray alphabet is shorter than the Morse

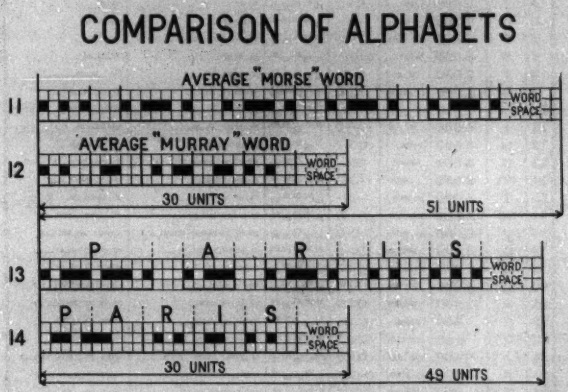

Fig. 7.

in the ratio of 30 to 49, or 38.78 per cent. Practically, the two alphabets are in the ratio of 3 to 5. Hence using the same number and length of current impulses in each case, a speed of 100 words a minute with the Murray alphabet would not be more than about 60 words a minute in Morse. The saving in the Murray alphabet lies chiefly in the fact that there is no space between the letters. Indeed, signals in adjoining letters not unfrequently coalesce, as will be seen in the case of the letters P and A in the word PARIS, where one current impulse is actually shared by the two letters, and five letters are transmitted by seven im-

pulses, whereas in the Morse representation of the same word no less than 14 impulses are required. Many combinations of letters like te, ta, ma, me, ou, are sent by one impulse, and the German name Latzke, consisting of six letters, requires only four current impulses for its transmission. This, however, is exceptional, the average being 1.25 impulses per letter. The Hughes printing telegraph employs only one impulse per letter, but it is extremely wasteful of the space between signals, so much so that, on the average, not more than three impulses per second are transmitted over a line which will easily convey thirty per second. The Murray alphabet, on the other hand, economizes spaces as well as signals to the utmost extent.

Tests of the System.

From time to time, as the system developed, careful tests were made of its capacity, both on loops of varying lengths and on circuits between cities. The speeds mentioned below were calculated on the basis of five letters to the word and a word-space equal to one letter. That is to say, the receiving tape fed through the perforator in one minute was measured to find the number of letters it contained, and this number was divided by six. In April and May, 1900, a series of tests were made between New York and Chicago. Working direct from Chicago to New York via Meadville without a repeater, a distance of 1050 miles by the route of the wire, the best speed attained was 77 words a minute. Working with a repeater at Meadville, much better results were achieved, 102 words a minute being easily attained. The wire used was copper, 208 lbs. and 4.5 ohms per mile. Duplex working was readily secured. Attempts were made to reach a speed of 114 words a minute. These were only partially successful, but the results gave promise of a speed of at least 120 words a minute in the future.

The most exhaustive tests of the system were made, from the 17th October to the 3d November, 1900, between Boston and New York. Two hundred ordinary commercial messages were taken without selection from the files in the operating room of the Postal Telegraph Company in New York. They consisted of 160 business telegrams (including 18 in cipher) and 40 domestic and social. They averaged 10.8 words in the paid portion of each message. Following the usual practice of counting single figures as words, they averaged about the usual rate of 30 words

per message; but counting figures as single letters, and counting all letters by measuring the transmitting tape and dividing by six, the average was about 26 words per message. These 200 messages were perforated in Wheatstone tape. A column press despatch from the New York *Herald*, containing 5988 letters and 1261 words, or an average of 4.75 letters per word, was also prepared in Wheatstone tape. This press despatch and the 200 commercial messages were transmitted from Boston to New York day after day at speeds varying from 60 to 96 words per minute. It was found that the apparatus worked with great accuracy, the whole 200 messages frequently coming through without error. At other times occasional errors occurred, owing to swinging wires and other familiar line troubles common to all telegraph systems. In addition to the 200 messages prepared beforehand in New York, a number of ordinary messages were perforated in Boston and transmitted to New York without difficulty, and almost without experience on the part of the Boston operators.

In regard to the number of messages transmitted per hour, in tests made when the apparatus was running at 61 words per minute, the 200 messages came through to New York in one hour 23 minutes. This is a speed of about 144 messages per hour, or more than three times the average rate of transmission by the Morse key, 40 messages per hour being regarded as a fair day's work for an average American operator. At the 96-word rate the messages came through at about 230 per hour. At the same rate the press despatch, with the last part repeated, making in all 1720 words, came through without an error in 16 minutes 42 seconds, or at the actual rate of about 103 words per minute, the words averaging only 4.75 letters each. A speed of 110 words per minute (115 actual words per minute) was tried, but at this rate not more than 200 or 300 consecutive words came through free from error. The inventor believes, however, that with increased skill and improved instruments, his system will ultimately reach a reliable speed of 120 words a minute. Working on a loop from New York to Albany and back, up one side of the Hudson River and down the other, speeds of 114 and even 118 standard words per minute have been successfully obtained. The length of the Postal Telegraph Company's lines from New York to Boston is about 290 miles, and the lines include from 20 to 30 miles of cable. Over this comparatively

short line the system did not require any readjustment for weather, which varied during the tests from clear and cold to dense fog and rain all the way between the two cities. Duplex working was perfect.

It will be seen from the foregoing tests that this system, working at the 60-word rate, has a capacity of 140 messages per hour. Cutting this down to 120 messages per hour to allow for corrections and delays, and working duplex, there is an output of 240 messages per hour, or 50 per cent. more than the Morse quadruplex can achieve. Working at the 96-word rate, the system has the capacity to transmit two and a half times as many telegrams as the quadruplex. No doubt these figures will be considerably reduced in commercial practice, but there is a wide margin for such reduction.

The tests have shown that, owing to the characteristic alphabet, distances of 1000 miles are not an obstacle, and the inventor is sanguine enough to believe that it will be possible, though not at present commercially practicable, to work between New York and San Francisco, a distance of 3000 miles, at a speed of at least 40 words a minute, or double the rate of manual transmission.

NOTE.

The figures upon which the comparison of alphabets is based have been obtained by means of the following table, which shows the average number of signals per letter employed in the "International" Morse alphabet and in the "Murray" alphabet:—

1	2	3	4	5	6
Letters	Frequency	Morse Signals	Products (1)	"Murray" Signals	Products (2)
e	140	1	140	1	140
t	100	1	100	1	100
a	90	2	180	1	90
i	90	2	180	1	90
n	80	2	160	1	80
o	80	3	240	1	80
s	80	3	240	2	160
r	70	3	210	2	140
h	60	4	240	2	120
d	50	3	150	2	100
l	50	4	200	2	100
u	45	3	135	1	45
c	40	4	160	1	40
m	30	2	60	1	30
f	30	4	120	2	60
w	25	3	75	2	50
y	25	4	100	3	75
p	24	4	96	2	48
b	20	4	80	2	40
g	20	3	60	2	40
v	15	4	60	1	15
k	8	3	24	1	8
q	6	4	24	2	12
j	5	4	20	2	10
x	5	4	20	2	10
z	5	4	20	1	5
	1193		3094		1688

$$\frac{3094}{1193} = 2.59 \qquad \frac{1688}{1193} = 1.41$$

This table is based on the average frequency of occurrence of letters of the alphabet as shown by a printer's "bill of type" as given in "Practical Printing," by Southward. This frequency is given in column 2. In column 3 is given the number of "International" Morse signals for each letter, and in column 4 is given the products obtained by multiplying columns 2 and 3 together. The sum of column 4 divided by the sum of column 2 gives the average number of signals per letter as 2.59. "American" Morse reckoned out in the same way gives an average of about 2.4 or about 8% less. But when the space allowance between signals, letters and words is reckoned in, this 8% advantage is reduced to somewhere about 5%. Column 5 and 6 give similar figures for the "Murray" alphabet, which proves to have an average of 1.41 signals per letter. This average is reduced in practice to about 1.25, owing to the fact that the current impulses in two adjoining letters frequently coalesce. Taking into consideration the frequency of letters, the International Morse alphabet contains on the

average 5 dots to 3 dashes. Hence as the average Morse letter contains 2.59 signals, the average Morse word of 5 letters contains approximately 6 dots, 4 dashes, and 5 fractions. This fraction is .59 of an average dot-dash. As there are on the average 5 dots to 3 dashes, and as 1 dash is equal to 3 dots, if we divide 14 by 8 we get the average dot-dash, which is in this way found to be equal to 1.75 dots. .59 of this is almost exactly 1 dot. The 5 fractions are, therefore, 5 dots. Hence the average International Morse word of 5 letters consists approximately of 6 dots, 4 dashes, and 5 dots (for the letter fractions), and the necessary spaces, namely, 1 dot-space between each letter signal, 3 dot-spaces between each letter, and 6 dot-spaces at the end of the word. The average word in the Murray alphabet consists of five letters, each letter containing 5 dot-spaces filled with various combinations of signals. The word space is of the same length as the letters, namely 5 dot spaces. Having these figures, it is possible to make diagrammatic representations of an average word in the International Morse and in the Murray alphabet. This is done in the foregoing diagram headed Comparison of Alphabets. Line 11 gives an average International word, and line 12 gives an average Murray word.

A paper to be presented at the 150th Meeting of the American Institute of Electrical Engineers, New York, January 25th, 1901.

[ADVANCE COPY SUBJECT TO REVISION.]

AIR-GAP AND CORE DISTRIBUTION.

The Magnetic Flux and its Effect upon the Regulation and Efficiency of Dynamo-Electric Machinery. Part III.

BY W. ELWELL GOLDSBOROUGH.

At the Omaha meeting of the Institute I presented for your consideration an outline of a method[1] for the determination of the surface density of the magnetic flux at the air line boundaries of masses of magnetic material, when such masses are subjected to the influence of known electro-magnetic forces.

In the present paper it is my purpose to elaborate the method referred to, and to present the results of further experimental researches in support of the accuracy of the theoretical treatment.

By way of an introduction to new matter it is well to review the concept of the interaction of the elements involved which is the basis upon which all amplifications of the method rest. This concept is simply but effectively developed by a consideration of such an arrangement of parts as is shown in Fig. 1. Here the armature is taken as a plane extending indefinitely to right and left. The pole-piece is a rectangular block extending indefinitely in a vertical direction above the armature, and having perfectly square corners. The length of the air-gap or the vertical distance from the pole-face to the armature surface is the same at all points. Both the armature and the pole-piece extend indefinitely at right angles to the plane of the paper, Fig. 1 being only a cross-sectional view of the apparatus assumed.

1. Transactions, vol. xv., pp. 515–529, 1898.

Suppose, now, that an M. M. F. is applied to the pole-piece to set up a magnetic field in the air-gap. Since the permeance of the pole-piece and of the armature is high, we can consider, for all practical purposes, that all points on the sides and face of the pole-piece are at the same potential relatively to the armature surface, and that all points on the armature surface are at the same potential relatively to the pole-piece, and that, therefore, a constant difference of magnetic potential exists between points on the pole-piece and points on the armature. That is, for all practical purposes, there is the same difference of potential between (a–o) as there is between (c–n) or (d–m).

And, *the number of lines of force that pass between any pair of points on the pole-piece and the armature respectively* (as n–d, in the plane of the paper) *is inversely proportional to the distance*[1]

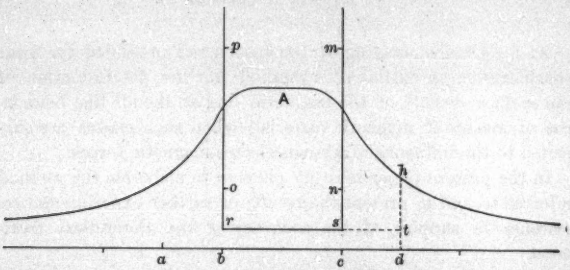

FIG. 1.

between them. And the strength of the magnetic field at the surface of the armature at any point (as d) *is proportional to the sum of the reciprocals of the distances of the point* (d) *from all the points on the perimeter of the pole-piece made by a section-plane passing through the point* (d), as in Fig. 1.

That this statement of the law of the variation of the strength of the magnetic field holds true under the limitations imposed, is readily apparent when it is remembered that the lines of magnetic force that radiate from any point on the line cut in the polar surface by the section-plane, may be considered as confined within the limits of the section-plane, and to create a

1. In measuring these distances it must be remembered that the most direct path *in air* must be taken, as (d–r–o) Fig. 1, *not* (d–o) direct.

field varying in its intensity inversely as the distances taken from the point. An infinite number of such section-planes, arranged parallel to one another and at right angles to the polar and armature surfaces, and, therefore, extending in a pile to infinite distances above and below the plane of the paper, will hold within their surfaces the magnetic lines emanating from the pole, for the magnetic forces at similarly situated points maintain such an equipoise by mutual repulsion as to prevent themselves from escaping the confines of their respective planes. For such cases the law of *inverse squares* does not hold, whereas, the law of *inverse proportion* does hold.

If the source of M. M. F. be brought within the more immediate vicinity of the armature surface, the conditions of potential distribution are at once modified. Suppose the energizing coil

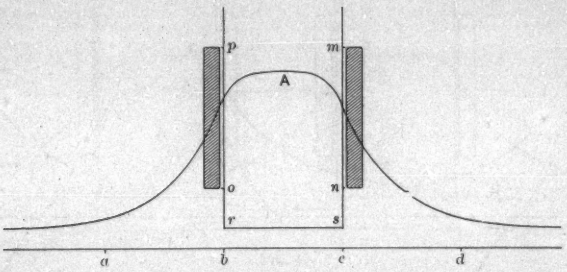

FIG. 2.

to occupy a position between (o–n) and (p–m). Then, as before, there will be a constant difference of potential between (a–o) and (c–n), but the potential difference between (d–m) will be zero. Under these conditions, then, the difference of potential between points on the polar surfaces below (o–n) and the armature is a maximum. The difference of potential between polar points above (p–m) and the armature surface is zero, and the potential between the armature and points on the polar surfaces (n–m) and (o–p) will be proportional to the distance the points are within the coil and above (o–n), divided by the height of the coil (n–m).

Therefore, *where potential variations occur, the number of lines of force that pass between any pair of points on the pole-*

piece and the armature respectively is directly proportional to the potential between the points, and inversely proportional to the distance between them. And the strength of the magnetic field at the surface of the armature at any point is proportional to the sum of the products of the differences of potential between the point and all the points on the perimeter of the pole-piece, by the respective reciprocals of the distances of the point from all the points on the perimeter of the pole-piece (as shown in Fig. 2).

Armature Surface Density.

The curve A in Fig. 1 represents the distribution of the magnetic flux at the *surface of the armature* as determined by the foregoing method; for the length of any ordinate (h–d) at any

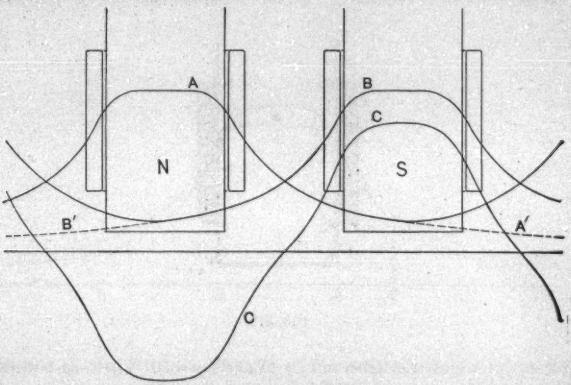

Fig. 3.

point (h) on this curve is equal to the sum of the reciprocals of the distances of the foot of the ordinate (d) from all points on the pole-piece. The curve A extends indefinitely to right and left, and is asymptotic to the surface of the armature. The curve indicates that the strength of the magnetic field does not diminish as rapidly as is usually supposed after the limits of the working air-gap proper are passed.

Fig. 2 illustrates the field strength at the surface of the armature when a pole is acting upon it which carries an energizing coil placed but slightly above the pole corners. The ordinates

of this curve are not so high as those of curve A, Fig. 1, on account of the difference in the potential distribution in the two cases. Suppose we bring up another pole, similar in every respect to the first, except that it is of opposite polarity. It will tend to set up a field density at the surface of the armature that is represented by curve B of Fig. 3; but since this magnetic field is opposed to that of the pole N, a field will result that is equal to the difference of the superimposed fields, or to the field strength represented by the curve C, Fig. 3. As a result of the subtraction, the flux density is shown to be zero at any point on the surface of the armature that is midway between the two poles, and that the sign of the field is reversed at such point. In this case the strength of the field *at the surface of the armature* is greatest at the center of the poles and diminishes to right and

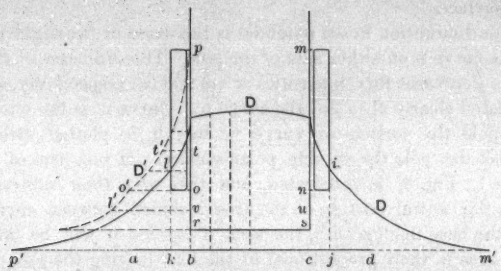

FIG. 4.

left. The reduction in the strength of the field under the pole corners is marked, and the form of the fringing field is clearly outlined. It is to be further remarked that the contour of the curve C of flux distribution changes its form as the distance between the poles N and S is varied.

POLE FACE SURFACE DENSITY.

The flux entering the surface of the armature from any one of a number of poles acting upon the armature has been shown to be proportional to the ordinates of the curve C, Fig. 3. Now this flux must of necessity largely emanate from the pole face that is immediately above the surface affected, and by applying the process of reciprocal summation from points on the polar perimeter, to all points on that part of the armature surface which is covered by the curve C, between two consecutive zero points, it is possible to develop the curve of pole face surface density. The results of such a summation are shown in Fig. 4.

The ordinates of the curve D represent intensities of the flux leaving successive points on the perimeter (p–r–s–m), due to the presence beneath the pole of a part (a–d) of the armature surface that is equal to the polar pitch. The curve D is plotted against the polar perimeter (p–r–s–m) developed into the straight line (p'–b–c–m'). The ordinate (j–i) represents the intensity of the flux leaving the pole tip at (u), the ordinate above (c) represents the intensity of the flux leaving the pole corner (r), and so on for other points. All of the flux that leaves the pole below the line (o–n) is acted on by the maximum M. M. F. between the pole and the armature. The flux leaving points on the pole above (o–n) is acted on by M. M. F's which diminish gradually until (p–m) is reached, at and above which there is no M. M. F. acting to produce a field, and therefore no flux leaving the surfaces.

This diminution in the potential is the cause of the slight peak in the curve D on either side of the pole. The ordinates of these peaks represent flux intensities at (o) and (n) respectively, as is indicated clearly at (o') of the curve D'. Curve D' is the same as the pole tip portion of curve D, only it is plotted directly against the pole tip or side polar surface. If one loop of the curve C, Fig. 3, is integrated, and this area then subtracted from the central portion of the area included between curve D and the base line (p'–m'), the area D–i–j–k–l–D will be taken. This area is then proportional to the flux leaving the pole and entering the armature, and as from any one elementary part of an area the flux can only flow in one direction, the polar limits of the area D–i–j–k–l–D should be pole tip points (u and v) above which none of the flux leaving the pole enters the armature.

Polar Surface Leakage Flux.

In the case of two electrically energized poles, such as poles N and S of Fig. 5, there is always a considerable flow of magnetic flux between them. If we consider the armature entirely removed the whole force of the M. M. F. acting will be expended in setting up a leakage field between the poles. In as much as the M. M. F. acting and producing this field is the resultant of two coils, instead of being due to one only, the difference of potential between adjacent pole faces will be twice that acting between any pole face and the armature. Furthermore, the difference of potential between the pole faces (s) and the yoke

(m') will be equal to the M. M. F. acting between the pole face and the armature, or to one-half the M. M. F. acting between adjacent pole faces. Therefore, by the application[1] of the method

1. NOTE.—By carrying out the application of the principles upon which the method depends to the limit, in the calculation of leakage, it will be found that any inconsistency which there may seem to be in the statement, "there will be an appreciable amount of leakage flux passing from the upper . . . polar surfaces," vanishes. To accomplish this, no surface between which and the point from which the summation is taken there is the least potential difference should be neglected. To be more explicit, let us refer to Fig. A, and consider the

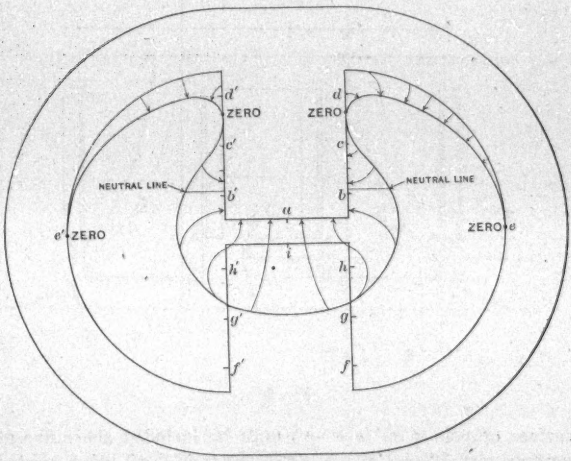

FIG. A.

flux entering the upper pole face at the point (a). There is evidently a positive difference of potential between (a) and every point on the perimeter (b–c–d–e–f–g–h–i–h'–g'–f'–e'–d'–c'–b') of the section of the cylindrical system shown in the figure, as the energizing coils occupy the spaces between (b–b'), (d–d') and (h–h'), (f–f'). Therefore the flux entering at (a) is proportional to the summation of reciprocals, taken with proper reference to M. M. F., from all parts of the perimeter. If we consider any other point, as for example (c), it is found that there is a positive potential between (c) and all points on the perimeter (c–d–e–f–g–h–i–h'–g'–f'–e'–d'–c') but a negative potential between (c) and all points on the perimeter (c–b–a–b'–c'). Therefore, the flux entering at any point below (b–b') as (c), is proportional to the summation of reciprocals, *taken with proper reference to* M. M. F. *and sign*. The result of a complete set of summa-

as outlined there will be an appreciable amount of leakage flux passing from the upper, as well as the lower polar surfaces, inclosed by the energizing coils.

The curve E of Fig. 5 shows approximately (see foot-note) the extent of the leakage field developed between the pole N and the pole S. The flux indicated by the curve E′ passes to the pole at the left of pole N. The complete amount of leakage flux from the pole N is represented, of course, by the combined areas included between the curves E and E′ and the polar perimeter.

In taking the summations for these leakage curves, the whole

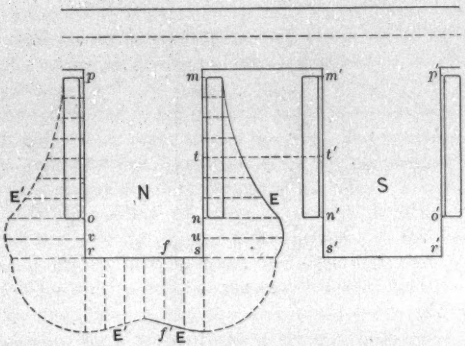

FIG. 5.

surface of pole S, $(m'-s'-r'-p')$ must be included, the different portions, $(m'-n')$, $(n'-s')$, $(s'-r')$, $(r'-o')$ and $(o'-p')$ being treated with proper reference to the M. M. F. acting between each and the point on the pole N from which the summation is taken.

tions taken with reference to M. M. F. and *sign* for a series of points is the development of such a flux distribution curve as is diagrammatically indicated about the upper pole and ring of the figure, which shows the development of zero flux points between (d–c) and (d′–c′) above and below which there is an exchange of flux between the poles sides and the ring, and the further development of two neutral points between (c–b) and (c′–b′), below which there is an interchange of flux directly between the poles. For practical purposes it does not seem necessary to extend the process of taking the summation beyond the limits of the pole of opposite sign, as the area of the leakage field curve so obtained practically equals the sum of the areas developed by the more elaborate application.

The Coefficient of Magnetic Leakage.

To determine the actual distribution of the flux over the polar surfaces, the leakage field of Fig. 5 must be superposed upon the pole face distribution curve of Fig. 4. This super-position is shown in Fig. 6. The points (*i*) and (*l*), at which the curves D and E cross, determine the limits of the useful and leakage fields of the system. The flux shown by those parts of the curve E which lie above (*l*–*v*) and (*u*–*i*) set the boundaries of the leakage field, while the parts of the curve D which lie below (*s*–*v*)

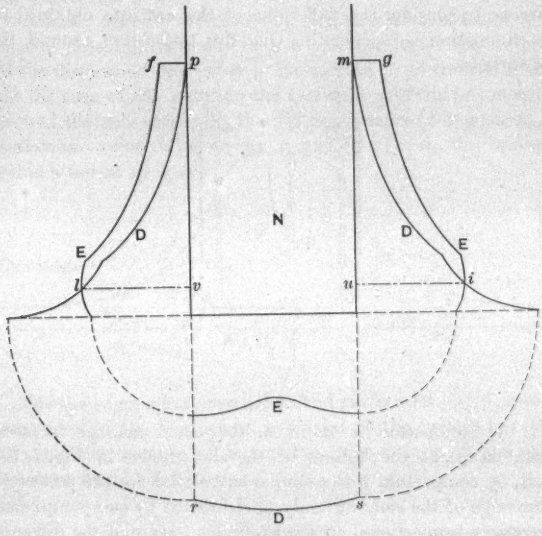

Fig. 6.

and (*u*–*i*) mark the extent of the useful field. It is, therefore, evident that the coefficient of magnetic leakage of the system will be equal to the ratio of the sum of the useful and leakage areas (*u*–*m*–*g*–*i*–*s*–*r*–*l*–*f*–*p*–*v*–*u*) to the useful area (*u*–*i*–*s*–*r*–*l*–*v*–*u*). By this combination of curves we have determined the final factor in the exploration of an electro-magnetic system that is composed of one or more poles acting upon a plane surfaced armature.

The Effect of Slots.

When slots or grooves are cut in the surface of the armature they diminish the total flux passing from the poles to the armature surface by a considerable amount. In making a calculation of the distribution of the flux over the bottom and sides of a slot the same method of reciprocal summation should be followed that is used to determine armature surface density curves, such as curve A of Fig. 2. However, owing to the fact that the sides of the slot materially diminish the extent of the approach usually open to flux as it passes from the pole to the armature, it is wrong to consider the full value of the ordinate obtained by this summation as representing the flux intensities around the slot surfaces.

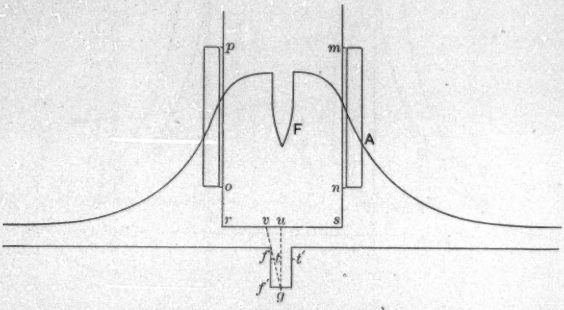

Fig. 7.

If the summation is taken in the usual manner for such a point as (g), at the bottom of the slot shown in Fig. 7, the result, by comparison with a similar summation for the point (f') at the edge of the slot, will only be decreased by an amount that is inversely proportional to the average increase of the distance of the point (g) from the pole surface points. If the confining influence of the sides of the slot was removed, this decrease would represent a true condition; but as the confining influence cannot be removed, it greatly reduces the permeance of the path open to the flux, and a contraction co-efficient has to be introduced to reduce the flux ordinate in proportion to the average decrease in the width of the approach imposed by the dimensions of the slot. By this means a correct estimate of the effect of the slots upon the permeance of the air-gap is made possible.

Mathematical Treatment.

The calculation of such flux distribution curves as have been shown in the preceding figures is best accomplished by an application of the calculus.

In developing a series of formulas to cover all the probable cases let us first turn to Fig. 1. In determining the value of the ordinate at (d) three operations are necessary. One integration will give the summation of reciprocals from (d) over the surface $(r\text{–}p)$, but a second must be taken for the surface $(s\text{–}r)$ and a third for the surface $(s\text{–}m)$.

Consider first the surface $(r\text{–}p)$, which is at a constant difference of potential relatively to (d). Let (y) denote the reciprocal of the distance of any point on the surface $(r\text{–}p)$ from (d), and x the distance of any point on the surface $(r\text{–}p)$ from (d) measured around the pole corner (r), then, if the distance $(d\text{–}r\text{–}p) = x_m$, the maximum value attained by x, and $(d\text{–}r) = x_n$, the minimum value attained by x, since

$$y = \frac{1}{x}$$

therefore

$$\sum_{x_n}^{x_m} y_{r-p} = \int_{x_n}^{x_m} \frac{dx}{x} = \log_e(x) \Big[_{x_n}^{x_m} = \log\left(\frac{x_m}{x_n}\right) \tag{1}$$

For the summation over the second surface let (y) denote the reciprocal of the distance of any point on the surface $(s\text{–}r)$ from (d), (x) the distance of any point on $(r\text{–}s)$ from (t), and (a) the constant length $(d\text{–}t)$, then

$$y = \frac{1}{\sqrt{a^2 + x^2}}$$

and if $x_m = (t\text{–}r)$ and $x_n = (t\text{–}s)$

$$\sum_{x_n}^{x_m} y_{s-r} = \int_{x_n}^{x_m} \frac{dx}{\sqrt{a^2 + x^2}} = \log_e\left(x + \sqrt{a^2 + x^2}\right) \Big[_{x_n}^{x_m}$$

$$= \log_e\left(\frac{x_m + \sqrt{a^2 + x^2{}_m}}{x_n + \sqrt{a^2 + x^2{}_n}}\right) \tag{2}$$

Since the summation over the surface $(s–m)$ is accomplished by the application of formula (2), in which the value of (a) is the distance $(d–o)$, therefore, making $x_m = (o–m)$ and $x_n = (o–s)$,

$$\sum_{x_n}^{x_m} y_{s-m} = \log_e \left(\frac{x_m + \sqrt{a^2 + x_m^2}}{x_n + \sqrt{a^2 + x_n^2}} \right) \qquad (2)$$

and the value of the ordinate at the point (d), or

$$(d - h) = \sum_{x_n}^{x_m} y_{r-p} + \sum_{x_n}^{x_m} y_{s-r} + \sum_{x_n}^{x_m} y_{s-m}$$

The ordinates of the other points on curve A, Fig. 1, are obtained, by the application of (1) and (2), in a similar way.

Where the energizing coil surrounds the pole as in Fig. 2, a modification has to be introduced in taking summations of reciprocals over the surfaces $(o–p)$ and $(n–m)$, owing to the variations in the M. M. F. within the coil. However, the surfaces $(r–o)$ and $(s–n)$ of Fig. 2, are treated in the same way as the surfaces $(r–p)$ and $(s–m)$ are treated in the case of Fig. 1.

In considering the summation from (d) to $(o–p)$, provision must be made for the fact that the M. M. F. between (o) and (d) is a maximum while the M. M. F. between (p) and (d) is zero. To do this a *proportional* M. M. F. *factor* must be introduced into the differential of (1), which has the value one when $x = (d–r–o) = x_n$, and the value of zero when $x = (d–r–p) = x_m$.

$\frac{A - x}{B}$, in which $A = x_m$ and $B = x_m - x_n$, or $\frac{x_m - x}{x_m - x_n}$, (11)

is such a factor, and therefore, in Fig. 2,

$$\sum_{x_n}^{x_m} y_{o-p} = \int_{x_n}^{x_m} \frac{A - x}{B} \frac{dx}{x} = \frac{A}{B} \log_e (x) - \frac{x}{B} \Big|_{x_n}^{x_m}$$

$$= \frac{x_m}{x_m - x_n} \log_e (x) - \frac{x}{x_m - x_n} \Big|_{x_n}^{x_m}$$

$$= \frac{x_m}{x_m - x_n} \log_e \left(\frac{x_m}{x_n}\right) - 1 \qquad (3)$$

In the case of the surface (n–m), Fig. 2, the same M. M. F. reduction factor is introduced into the differential of (2) as in the case just treated. Then,

$$\sum_{x_n}^{x_m} y_{n-m} = \int_{x_n}^{x_m} \frac{A - x}{B} \frac{dx}{\sqrt{a^2 + x^2}} = \frac{A}{B} \log_e (x + \sqrt{a^2 + x^2}) -$$

$$\frac{\sqrt{a^2 + x^2}}{B} \Bigg]_{x_n}^{x_m}$$

$$= \frac{x_m}{x_m - x_n} \log_e (x + \sqrt{a^2 + x^2}) - \frac{\sqrt{a^2 + x^2}}{x_m - x_n} \Bigg]_{x_n}^{x_m}$$

$$= \frac{x_m}{x_m - x_n} \log_e \left(\frac{x_m + \sqrt{a^2 + x^2_m}}{x_n + \sqrt{a^2 + x^2_n}}\right) - \frac{\sqrt{a^2 + x^2_m} - \sqrt{a^2 + x^2_n}}{x_m - x_n} \qquad (4)$$

In determining the value of the ordinates of the pole face flux distribution curve the formulas so far developed only suffice in part. The length of such an ordinate as (o–o') of Fig. 4, is determined by two operations, one giving the summation over the surface (a–b) and the other the summation over (b–d). In the former case formula (2) is used, (a) being made equal to the distance (o–b) and x varying in value from zero at (b) to (b–a) at (a).

To effect the second summation a new expression is necessary, as the distance (o–r) enters as an independent constant in determining each reciprocal. Let x equal the distance from (b) of any point on (b–d), (a) equal the constant distance (r–b) and (b) the constant distance (o–r), then, putting $x_m = (b\text{–}d)$, and $x_n =$ zero,

$$y = \frac{1}{b + \sqrt{a^2 + x^2}}$$

and

$$\sum_{x_n}^{x_m} y_{b-d} = \int_{x_n}^{x_m} \frac{dx}{b + \sqrt{a^2 + x^2}}$$

therefore, when $a > b$:-

$$\sum_{x_n}^{x_m} y_{b-d} = \log_e (x + \sqrt{a^2 + x^2}) - \frac{2b}{\sqrt{a^2 - b^2}} \tan^{-1}\left(\frac{x + b + \sqrt{a^2 + x^2}}{\sqrt{a^2 - b^2}}\right)\Bigg]_{x_n}^{x_m}$$

$$= \log_e\left(\frac{x_m + \sqrt{a^2 + x^2_m}}{x_n + \sqrt{a^2 + x^2_n}}\right) - \frac{2b}{\sqrt{a^2 - b^2}}\left[\tan^{-1}\left(\frac{x_m + b + \sqrt{a^2 + x^2_m}}{\sqrt{a^2 - b^2}}\right)\right.$$

$$\left. - \tan^{-1}\left(\frac{x_n + b + \sqrt{a^2 + x^2_n}}{\sqrt{a^2 - b^2}}\right)\right] \qquad (5)$$

when $a = b$:-

$$\sum_{x_n}^{x_m} y_{b-d} = \log_e (x + \sqrt{a^2 + x^2}) + \frac{2a}{x + a + \sqrt{a^2 + x^2}}\Bigg]_{x_n}^{x_m}$$

$$= \log_e\left(\frac{x_m + \sqrt{a^2 + x^2_m}}{x_n + \sqrt{a^2 + x^2_n}}\right) + \frac{2a}{x_m + a + \sqrt{a^2 + x^2_m}}$$

$$- \frac{2a}{x_n + a + \sqrt{a^2 + x^2_n}} \qquad (6)$$

and when $a < b$:-

$$\sum_{x_n}^{x_m} y_{b-d} = \log_e (x + \sqrt{a^2 + x^2})$$

$$- \frac{b}{\sqrt{b^2 - a^2}} \log_e\left(\frac{x + \sqrt{a^2 + x^2} + b - \sqrt{b^2 - a^2}}{x + \sqrt{a^2 + x^2} + b + \sqrt{b^2 - a^2}}\right)\Bigg]_{x_n}^{x_m}$$

$$= \log_e\left(\frac{x_m + \sqrt{a^2 + x^2_m}}{x_n + \sqrt{a^2 + x^2_n}}\right)$$

$$-\frac{b}{\sqrt{b^2-a^2}}\log_e\left[\left(\frac{x_m+\sqrt{a^2+x_m^2}+b-\sqrt{b^2-a^2}}{x_m+\sqrt{a^2+x_m^2}+b+\sqrt{b^2-a^2}}\right)\right.$$

$$\left.\left(\frac{x_n+\sqrt{a^2+x_n^2}+b+\sqrt{b^2-a^2}}{x_n+\sqrt{a^2+x_n^2}+b-\sqrt{b^2-a^2}}\right)\right] \qquad (7)$$

And, therefore, since in the present case of point (o), (a) is less than (b), formula (7) gives the desired summation over $(b\text{–}d)$.

For the summation over this surface $(b\text{–}d)$ from (v), formula (6) is used, as $(v\text{–}r) = b = (r\text{–}b) = a$. For points between (v) and (r) formula (5) would be used, as (a) is greater than (b) for such points.

When estimating the intensity of the flux leaving points on the side of the pole above $(o\text{–}n)$, the value of the proportional ordinate is first obtained in the usual way, and then reduced in direct proportion to the distance the explored point is from the top of the coil. The value of the ordinate to curve D′ at (t), Fig. 4, is equal to,—

$$(t-t') = \left[\sum_{x_n}^{x_{in}} y_{b-a} \text{ by (2)} + \sum_{x_n}^{x_m} y_{b-d} \text{ by (7)}\right]\frac{(t-p)}{(o-p)} \qquad (14)$$

where $x_m = (b\text{–}a)$ in (2), and $(b\text{–}d)$ in (7), and $x_n = o$ in both (2) and (7). In a case such as is shown in Fig. 4, formulas (2), (5), (6) and (7) are all that are needed.

In the determination of data for plotting leakage flux curves, like those shown in Fig. 5, other forms are encountered. The value of the flux ordinate at (s) is determined by the application of (2) to $(s'\text{–}n')$ and (1) to $(s'\text{–}r'\text{–}o')$, each result being doubled, as there is double the pressure between the poles that there is between a pole and the armature. For the summations over $(n'\text{–}m')$ and $(o'\text{–}p')$, however, special proportional M. M. F. factors must be used as the proportional value of the M. M. F. between (s) and $(n'\text{–}o')$ is 2, and between (s) and $(m'\text{–}p')$ is one.

To find the value of the proportional M. M. F. factor, put $x_n = (s'\text{–}n')$ and $x_m = (s'\text{–}m')$ in,—

$$\frac{A-x_n}{B} = 2 \text{ and } \frac{A-x_m}{B} = 1$$

then

$$B = \frac{A - x_n}{2}, \text{ and also, } B = A - x_m$$

therefore $$A - x_n = 2A - 2x_m$$

or $$A = 2x_m - x_n$$

and $$B = x_m - x_n$$

and the value of the proportional M. M. F. factor is

$$\frac{A - x}{B} = \frac{2x_m - x_n - x}{x_m - x_n} \tag{12}$$

Now writing A $= (2x_m - x_n)$, and B $= (x_m - x_n)$ in (4) and (3), (4) and (3) can be applied directly to the obtaining of complete summations over the surfaces $(n'\text{–}m')$ and $(o'\text{–}p'$, respectively. The $[y_{(n'-m')}]$ and $[y_{(o'-p')}]$ values so determined must not be doubled before being added to the results obtained above for the surfaces $(s'\text{–}n')$ and $(s'\text{–}r'\text{–}o')$, as such correction is taken care of by the proportional M. M. F. multiplier of (12).

In obtaining summations for points such as (t), located on the pole surface within the coil, special care must be taken to provide correct proportional M. M. F. factors. For the surfaces $(n'\text{–}s')$ and $(s'\text{–}r'\text{–}o')$, to which (2) and (1) apply respectively, constant value proportional M. M. F. factors

$$\text{equal to } \frac{(t - m) + (m' - n')}{(m - n) + (m' - n')} \text{ must be used.} \tag{13}$$

For the surface $(o'\text{–}p')$ the variable proportional M. M. F. factor must range in value from

$$\frac{(t - m) + (m' - n')}{(m - n) + (m' - n')}, \text{ when } x = (t - s' - r' - o') = x_n, \text{ to}$$

$$\frac{(t - m)}{(m - n) + (m' - n')}, \text{ when } x = (t - s' - r' - p') = x_m \text{ and}$$

be applied in (3).

Similarly the proportional M. M. F. factor for the surface

$(t' - m')$ must vary between $\dfrac{(t - m) + (t' - m')}{(m - n) + (m' - n')}$ and

$\frac{t-m}{(m-n)+(m'-n')}$ when $x = x_n$ = zero, and $x = (t'-m') = x_m$, respectively.

The limits of the value of the factor for the surface $(t'-n')$ are $\frac{(t-m)+(n'-m')}{(n-m)+(n'-m')}$ and $\frac{(t-m)+(t'-m')}{(n-m)+(n'-m')}$ for $x = (t'-n') = x_m$ and $x = x_n$ = zero, respectively. The two last mentioned factors are used with (4), (a) being equal to $(t-t')$.

There are certain other cases which arise which can only be met by the introduction of a proportional M. M. F. factor into the differential of (5), (6) and (7). Such a case is well illustrated by a determination of the leakage flux ordinate of the curve E at the point (f'), Fig. 5. The summation over the surface $(s'-r'-o')$ is easily effected from (f') by applying (1) and doubling the result. The summation over the surface $(o'-p')$ is effected by applying (3) in connection with (12). The summation over $(s'-n')$ is obtained by applying (5) and doubling the result. But the summation from (f'), Fig. 5, over $(n'-m')$ can only be calculated by a proper substitution in the integral of the product of (12) by the differential of (5).

Here then, we have,—

$$y = \frac{A-x}{B} \frac{dx}{b + \sqrt{a^2+x^2}}$$

and

$$\sum_{x_n}^{x_m} y_{(n'-m')} = \int_{x_n}^{x_m} \frac{A-x}{B} \frac{dx}{b + \sqrt{a^2+x^2}}$$

when $a > b$

$$= \frac{1}{B} \Big\{ [A \text{ times formula (5)}] - [\sqrt{a^2+x^2} - b \log_e (b + \sqrt{a^2+x^2})] \Big\} \Big[\Big]_{x_n}^{x_m}$$

$$= \frac{A}{B} \Big\{ \log_e \left(\frac{x_m + \sqrt{a^2+x_m^2}}{x_n + \sqrt{a^2+x_n^2}} \right) -$$

$$\frac{2b}{\sqrt{a^2-b^2}}\left[\tan^{-1}\left(\frac{x_m+b+\sqrt{a^2+x^2_m}}{\sqrt{a^2-b^2}}\right)-\tan^{-1}\left(\frac{x_n+b+\sqrt{a^2+x^2_n}}{\sqrt{a^2-b^2}}\right)\right]\Bigg\}$$

$$-\left\{\frac{\sqrt{a^2+x^2_m}-\sqrt{a^2+x^2_n}}{B}-\frac{b}{B}\log_e\left(\frac{b+\sqrt{a^2+x^2_m}}{b+\sqrt{a^2+x^2_n}}\right)\right\} \quad (8)$$

when $a = b$

$$= \frac{1}{B}\Big\{[A \text{ times formula (6)}] -$$

$$[\sqrt{a^2+x^2}-b\log_e[b+\sqrt{a^2+x^2}]]\Big\}\Big[_{x_n}^{x_m}$$

$$= \frac{A}{B}\left\{\log_e\left(\frac{x_m+\sqrt{a^2+x^2_m}}{x_n+\sqrt{a^2+x^2_n}}\right)+\frac{2a}{x_m+a+\sqrt{a^2+x^2_m}}-\right.$$

$$\left.\frac{2a}{x_n+a+\sqrt{a^2+x^2_n}}\right\}-\left\{\frac{\sqrt{a^2+x^2_m}-\sqrt{a^2+x^2_n}}{B}-\right.$$

$$\left.\frac{b}{B}\log_e\left(\frac{b+\sqrt{a^2+x^2_m}}{b+\sqrt{a^2+x^2_n}}\right)\right\} \quad (9)$$

when $a < b$

$$= \frac{1}{B}\Big\{[A \text{ times formula (7)}] -$$

$$[\sqrt{a^2+x^2}-b\log_e(b+\sqrt{a^2+x^2})]\Big\}\Big[_{x_n}^{x_m}$$

$$= \frac{A}{B}\left\{\log_e\left(\frac{x_m+\sqrt{a^2+x^2_m}}{x_n+\sqrt{a^2+x^2_n}}\right)-\right.$$

$$\frac{b}{\sqrt{b^2-a^2}}\log_e\left[\left(\frac{x_m+\sqrt{a^2+x^2_m}+b-\sqrt{b^2-a^2}}{x_m+\sqrt{a^2+x^2_m}+b+\sqrt{b^2-a^2}}\right)\right.$$

$$\left.\left.\left(\frac{x_n+\sqrt{a^2+x^2_n}+b+\sqrt{b^2-a^2}}{x_n+\sqrt{a^2+x^2_n}+b-\sqrt{b^2-a^2}}\right)\right]\right\}-$$

$$\left\{\frac{\sqrt{a^2+x^2_m}-\sqrt{a^2+x^2_n}}{B}-\frac{b}{B}\log_e\left(\frac{b+\sqrt{a^2+x^2_m}}{b+\sqrt{a^2+x^2_n}}\right)\right\} \quad (10)$$

Now, since in the case of the point (f') considered, (b) = (f–s) and is therefore less than (a) = (s–s'), formula (8) is used; and by putting $A = (2\,x_m - x_n)$, $B = (x_m - x_n)$, $x_m = (s' - m')$ and $x_n = (s' - n')$ in (8) and solving, the value of $[y_{n'-m'}]$, for (f') of Fig. 5, is readily found.

Finally, therefore, the ordinate (f-f') of curve E, Fig. 5, equals

$$(f - f') = \sum_{x_n}^{x_m} (2\,y_{(s'-r'-o')} + y_{(o'-p')} + 2\,y_{(s'-n')} + y_{(n'-m')}) \quad (15)$$

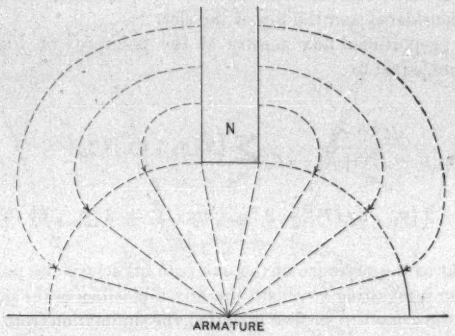

FIG. 8.

It has been pointed out (see page 34) that, to obtain the flux distribution over the side and bottom surfaces of slots a contraction co-efficient must be entered as a factor in the final evaluation of the reciprocal summation of all points lying within such slots. This co-efficient is given a value that is proportional to the reduction which the slot imposes upon the approach to a point in the slot from the pole. If a point is taken on a smooth armature surface the width of the approach from any surface that is (r) units distant above the point is ($\pi\,r + k$), where (k) is the width of the surface at the distance (r) above the point. That this assumption is justified is shown by a consideration of Fig. 8. But if a point, as point (g), Fig. 7, is taken at the bottom of a slot which has a depth (r) and a width (k), the approach to the point at a distance (r) from the point has now a width of only (k); and

since all the lines of force which pass from the pole to the point (g) have to pass between the edges of the slot opening, the slot diminishes the approach to the point in ratio of (k) to ($\pi r + k$).

In making calculations for the intensity of the flux entering the surface points of the slot the contraction coefficient is given the form

$$\text{contraction coefficient} = \frac{k}{\pi r + k} \tag{16}$$

in which (r) is the distance that the point in the slot is from the top of the slot, and (k) is the least width of the slot between the point considered and the top of the slot.

The proportional flux density at the point (g) of Fig. 7 is, therefore, equal to

$$\beta_g = \frac{(t - t')}{\pi (f - f') + (t - t')} \sum_{x_n}^{x_m} \Big(2\,[y_{u-r} \text{ by } (2)] + 2\,[y_{r-r} \text{ by } (7)] + 2\,[y_{r-o} \text{ by } (1)] + 2\,[y_{o-p} \text{ by } (3)] \Big) \tag{17}$$

in so far as the presence of the one pole influences the point.

When integrating to obtain the flux intensities at the pole face where the armature surface is slotted, the summation from the top surfaces of the teeth is obtained just as the surface summation for curve D of Fig. 4 was obtained. When the integration is carried to the sides and bottoms of the slots, however, variable contraction coefficients must be introduced which have the general form of the proportional M. M. F. factors discussed in connection with (12). For instance, if it is desired to obtain the summation at point (v), Fig. 7, from the surface (f-f') of the side of the slot, a contraction coefficient must be applied to the differential of formula (2) that has a value of

$$\frac{(t - t')}{(t - t')} = 1, \text{ when } y = \frac{1}{(u - f')} \tag{18}$$

and a value of

$$\frac{(t - t')}{\pi (f - f') + (t - t')}, \text{ when } y = \frac{1}{(u - f')} \tag{19}$$

Therefore put

$$\frac{A - x_n}{B} = 1$$

and

$$\frac{A - x_m}{B} = \frac{(t - t')}{\pi (f' - f'') + (t - t')},$$

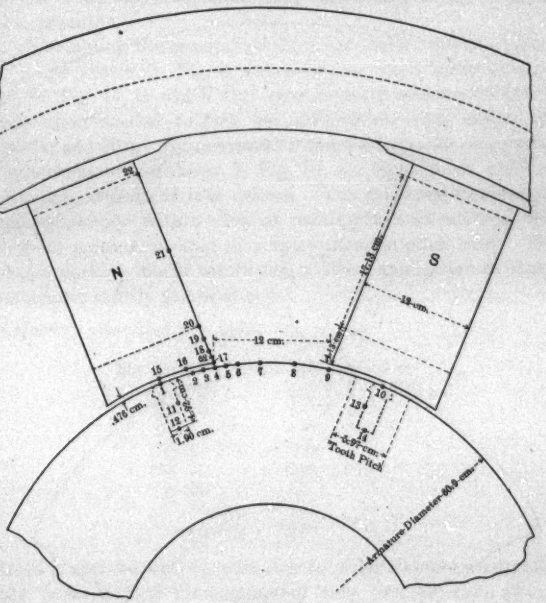

FIG. 9.

solve for the values of A and B and substitute them in (4). Then (4) applied to the case in hand will give the reciprocal summation at (u) covering the surface (f'–f''). Other points on the pole face relatively to other surfaces in the slots must be treated substantially in the same manner.

PRACTICAL APPLICATION.

In illustrating the practical application of the method that has been discussed to the determination of the flux distribution in

dynamo machines, I will take as a subject for discussion the 8–50–900–1150, two-phase alternator which was briefly referred to in the paper[1] read at the Omaha meeting. This machine is well suited to test the practical utility of such calculations, as it is representative of modern practice in alternator design. Furthermore, as the ratio of the circumferential width of one of its poles to the length of the pole taken parallel to the shaft is as

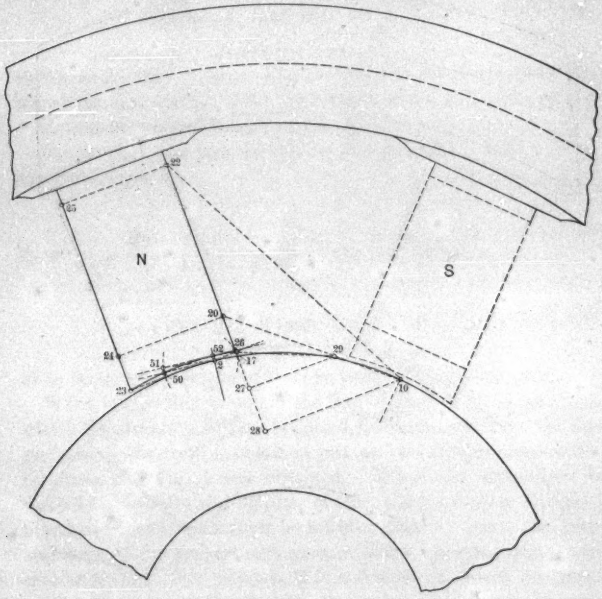

FIG. 10.

one is to two, a fair approximation to the assumptions made on page 25 are obtained in the plane which cuts through the center lines of the pole faces at right angles to the shaft of the machine.

1. The calculations reported at the Omaha meeting in so far as they refer to this generator are not carried out with the refinement that is here introduced, and are not therefore exact in anything like the same measure. The approximate methods there discussed were the only ones used, whereas every refinement that is mentioned in this paper has been applied in the present case.

An outline of a part of the field ring and armature of the generator is given in Fig. 9, together with the principal dimensions. The small numbered circles show the positions of the points for which the field intensities have been calculated. The number of points taken, twenty-three in all, amply provide for the complete exploration of the magnetic circuit, and their location is an indication of the sections to which it is most important to devote attention.

In determining the armature surface distribution curve, similar to the one shown in Fig. 2, measurements were taken as indicated in Fig. 10, in which the exact location is shown of every measurement needed to find the proportional flux density at points (2) and (10). By reference to the Table (1) the lengths of the construction lines shown in Fig. 10, can be obtained, and the calculations checked, if it is desired. The table and figures are presented simply with a view of making such calculations less tedious to persons wishing to employ them in other cases. By having a working model before one, a direct application in other cases is more readily followed out.

Referring specifically to point (2),

line	(2)–(52)	— .476	centimeters
"	(52)–(26)	2.41	"
"	(2)–(17)	— 2.46	"
"	(2)–(17)–(20)	— 6.59	"
"	(2)–(17)–(22)	— 23.72	"
"	(52)–(51)	— 5.33	"
"	(2)–(50)	— 5.37	"
"	(2)–(23)–(24)	— 13.73	"
"	(2)–(23)–(25)	— 30.86	"

Here it may be well to note that in order to more accurately apply formula (2) to the summation over line (50)–(52), an arc was described from point (2) as a center, with a radius equal to (2)–(50), cutting line (51)–(52), [which is at right angles to line (2)–(52), drawn through (2) and the center of the armature] at (51). Line (52)–(51) was then put equal to (x_m) in formula (2), instead of either line (2)–(50) or (52)–(50).

The same construction was used in obtaining the length (52)–(26), the line (2)–(17) being the radius in this case.

The data given for the treatment of point (10) can also be readily understood by following the graphical construction of Fig. 10. In this case

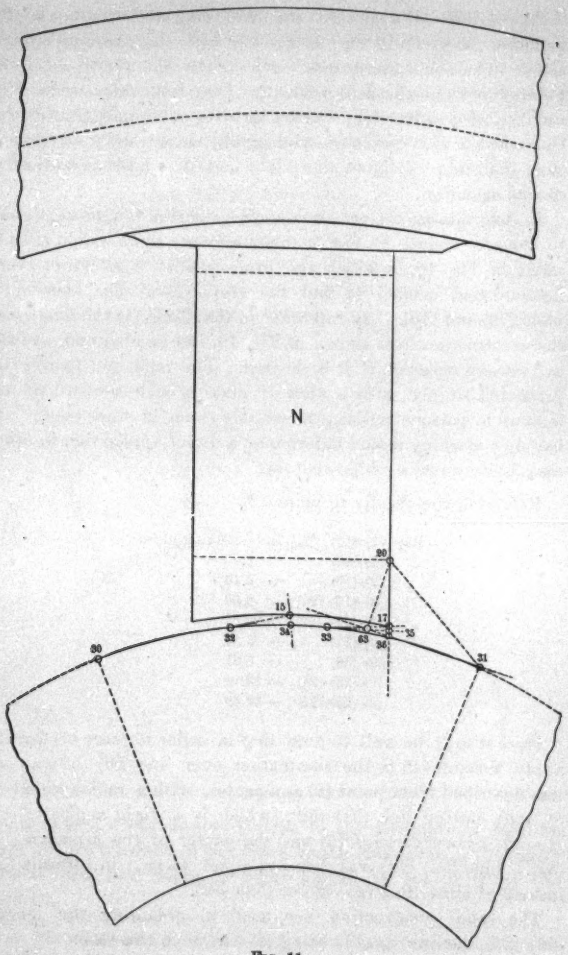

Fig. 11.

line (27)–(29)	= 9.80	centimeters
" (29)–(10)	= 7.40	"
" (27)–(17)	= 4.06	"
" (27)–(20)	= 8.20	"
" (10)–(28)	= 15.50	"
" (28)–(20)	= 12.95	"
" (28)–(22)	= 30.08	"
" (10)–(29)–(17)	= 18.00	"
" (10)–(29)–(2)–(23)–(24)	= 34.13	"
" (10)–(29)–(2)–(23)–(25)	= 51.26	"

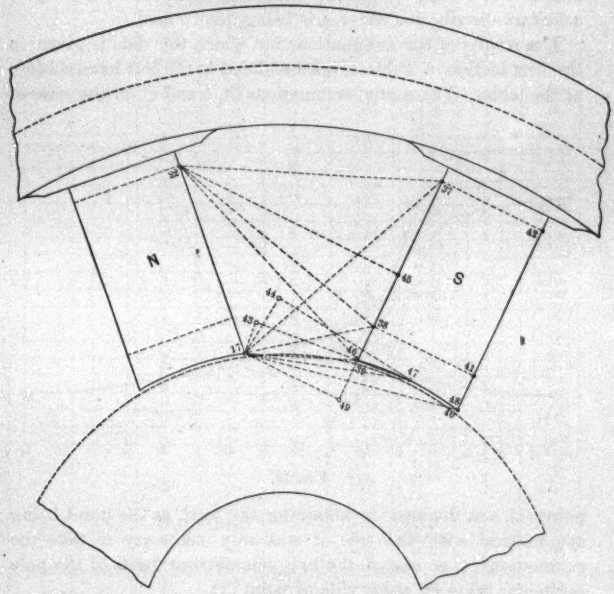

FIG. 12.

It is perfectly proper to produce constructions into the armature core and pole pieces, and take measurements therefrom so long as none of the distance lines, such as line (10)–(29)–(17), which are included in a summation fall materially inside of the boundaries of such portions of the iron part of the machine as are depended upon for the data. For this reason it is advisable to use formula (5) as illustrated, in getting the summation over the

surface (17)–(20), instead of using formula (2) and putting the distance (10)–(28)=(a) and the distances (28)–(17) and (28)–(20) respectively equal to (x_n) and (x_m) for such a summation. The latter procedure would require the first distance line to pass directly from (10) to (17), and thereby cut into a surface, all points of which are taken as being at a constant potential. Theoretically this is an impossibility.

In Fig. 10 the s pole is shown by a dotted line, as it is supposed to be removed at the time the data for the single pole armature distribution curve A is being determined.

The results of the summations for which the data is given in the first section of Table 1, are tabulated in the left hand column of the table. The partial summations (a, b and c), in the case of

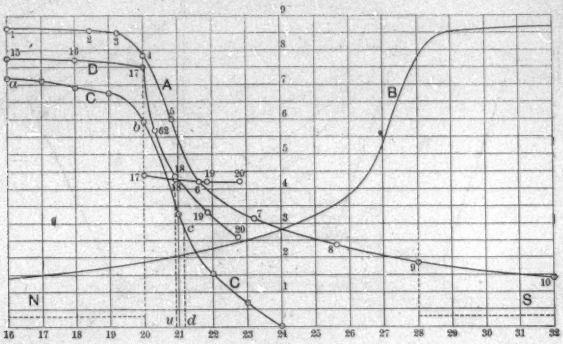

FIG. 13.

point (1), are doubled in obtaining the total, as the point being symmetrical with the pole it was only necessary to take the summations over one of the two symmetrical parts of the pole perimeter lying on either side of point (1).

Plotting the "totals" of points from (1) to (10) of Table 1, opposite properly spaced ground line divisions the curve A of Fig. 13 is obtained. As the data also suffices for plotting curve B of Fig. 13, which shows the field due to the s pole, it also appears. Subtracting the ordinates of curve B from those of curve A, curve C, Fig. 13 results, and shows the actual variations in the strength of the magnetic field along the surface of the armature, when there are no slots cut in the armature surface. Or, if there

TABLE I, SECTION 1.

SINGLE POLE ARMATURE SURFACE DISTRIBUTION CURVE DATA.

Number of figure from which data was obtained	Number of point from which summation is taken.	Data designation letter for separate integrations.	a	b	x_n	x_m	Number of formula used in integrating.	Partial summation values.
		a	.476	0	0	6.05	2	3.241
	1	b	0	0	6.05	10.18	1	.518
		c	0	0	10.18	27.31	3	.581
		total						8.680
		a	.476	0	0	2.41	2	2.315
10	2	b	0	0	2.46	6.59	1	.986
		c	0	0	6.59	23.72	3	.770
		d	.476	0	0	5.33	2	3.110
		e	0	0	5.37	13.73	1	.940
		f	0	0	13.73	30.86	3	.460
		total						8.581
		a	.476	0	0	1.22	2	1.634
	3	b	0	0	1.30	5.43	1	1.431
		c	0	0	5.43	22.56	3	.880
		d	.476	0	0	5.33	2	3.110
		e	0	0	5.37	14.73	1	1.00
		f	0	0	14.73	31.86	3	.436
		total						8.504
		a	0	0	.476	4.61	1	2.272
	4	b	0	0	4.61	21.74	3	.972
		c	.476	0	0.	5.33	2	3.110
		d	0	0	5 37	15.93	1	1.087
		e	0	0	15 93	33.06	3	.414
		total						7.855
		a	.455	0	1.1	5.42	2	1.510
	5	b	1.06	0	.72	4 85	2	2.038
		c	1.06	0	4.85	21.98	4	.920
		d	0	0	5.45	17.13	1	1.145
		e	0	0	17.13	34.26	3	.380
		total						5.993
		a	.37	0	2.42	5.42	2	.802
	6	b	2 22	0	1.73	5.86	2	1.000
		c	2.22	0	5.86	22.09	4	.834
		d	0	0	5.45	18.33	1	1.212
		e	0	0	18.33	35.46	3	.364
		total						4.212
		a	4.50	0	1.75	5.87	2	.703
	7	b	4.50	0	5.87	23.00	4	.646
		c	.1	0	4.82	20.73	2	1.458
		d	0	0	20 73	37.86	3	.334
		total						3.141
		a	7.76	0	3.16	7.29	2	.438
	8	b	7.76	0	7.29	24.42	4	.590
		c	0	0	8.40	24.23	1	1.058
		d	0	0	24.23	41.36	3	.290
		total						2.376
		a	10.86	0	5.00	9.13	2	.316
	9	b	10.86	0	9.13	26.26	4	.480
		c	0	0	12.00	27 88	1	.841
		d	0	0	27.88	45.01	3	.250
		total						1.887
		a	9.80	7.40	4.06	8.20	5	.220
		b	15.30	0	12.95	30.08	4	.350
10	10	c	0	0	18.00	34.13	1	.636
		d	0	0	34.13	51.26	3	.910
		total						1.416

NOTE: Point No. 1, being symmetrically placed with reference to pole N, the summation is only taken over half the polar surface. Values a, b and c, must therefore be doubled to obtain the "total."

TABLE I, SECTION 9.

Pole Face Density Distribution Curve Data.

Number of figure from which data was obtained	Number of point from which summation is taken.	Data designation letter for separate integration.	a	b	x_n	x_m	Number of formula used in integration.	Partial and total summation values.
		a	.476	0	0	3.61	2	2.721
11	15	b	0	0	3.69	12.00	1	1.180
		total						7.802
		a	.476	0	0	3.61	2	5.442
	16	b	0	0	3.69	9.00	1	.891
		c	0	0	3.69	15.00	1	1.403
		total						7.736
		a	.476	0	0	3.61	2	5.442
	17	b	0	0	3.69	6.00	1	.488
		c	0	0	3.69	18.00	1	1.585
		total						7.515
		a	.86	0	.17	6.16	2	2.352
	62	b	.476	.476	0	3.61	6	1.858
		c	0	0	4.17	18.48	1	1.488
		total						5.678
		a	1.72	0	.41	6.40	2	1.790
	28	b	.476	1.377	0	3.61	7	1.223
		c	0	0	5.07	19.38	1	1.341
		total						4.354
		a	3.03	0	.83	6.80	2	1.279
	19	b	.476	2.733	0	3.61	7	.850
		c	0	0	6.44	20.75	1	1.170
		total						3.299
		a	4.33	0	1.90	7.16	2	.963
11	20	b	.476	4.13	0	3.61	7	.572
		c	0	0	7.82	22.13	1	1.041
		total						2.576

Note: Point No. 11, being symmetrically placed with reference to pole N, the summation is only taken over half the polar surface.

TABLE I, SECTION 8.

Leakage Field Distribution Curve Data.

		a	11.15	0	4.00	8.73	2	.316
		b	11.15	0	8.73	25.86	4	1.338
		c	3.36	0	11.55	17.55	2	.406
12	17	d	7.92	0	16.50	22.50	2	.286
		e	0	0	23.60	27.70	1	.157
		f	0	0	27.70	44.80	3	.743
		total						4.411
		a	12.13	0	3.61	7.74	2	.293
		b	12.13	0	7.74	24.87	4	1.315
		c	2.30	0	12.40	18.40	2	.385
	18	d	6.04	0	17.60	23.60	2	.297
		e	0	0	24.40	28.50	1	.157
		f	0	0	28.50	45.60	3	.735
		total						4.304
		a	13.10	0	2.68	6.81	2	.288
		b	13.10	0	6.81	23.94	4	1.349
		c	0	0	13.30	20.40	1	.793
	19	d	0	0	20.40	46.50	3	.697
		total						4.908
		a	14.07	0	1.70	5.83	2	.281
		b	14.07	0	5.83	22.96	4	1.417
		c	0	0	14.10	30.30	1	.765
	20	d	0	0	30.30	47.40	3	.694
		total						4.903
		a	20.20	0	0	4.42	2	.314
		b	20.20	0	0	16.00	4	.779
		c	0	0	20.60	36.80	1	.578
	21	d	0	0	36.80	53.90	3	.396
		total						2.363
		a	26.20	0	0	10.80	4	.128
		b	26.20	0	0	6.30	4	.197
12	22	c	26.20	0	6.30	10.40	2	.147
		d	0	0	28.20	44.40	1	.449
		e	0	0	44.40	61.50	3	.170
		total						1.091

are slots in the armature surface, it shows the flux intensities at the tops of the teeth, by projecting them up to the curve, in

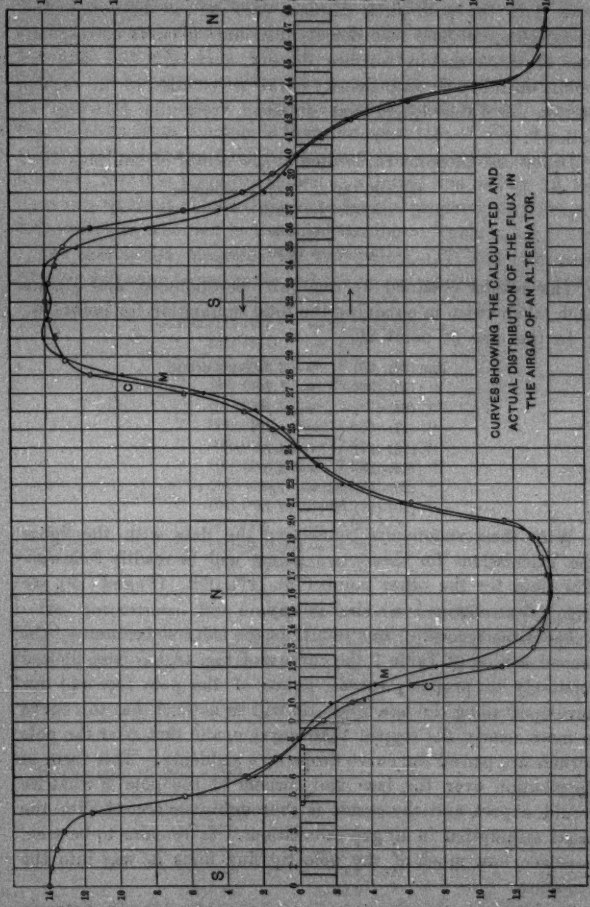

Fig. 14.

whatever positions they may be. A complete cycle of the armature surface density curve c is shown in Fig. 14.

Fig. 11 shows the graphical constructions for obtaining data for the pole face density ordinates at points (15) and (20). As in the case of curve D of Fig. 4, the summation here is taken over only the pole pitch distance (30)–(31), as this summation gives the actual pole face density distribution curve corresponding to the armature surface density curve C. The data for points on the pole face density curve is recorded in the second section of Table 1, and was obtained from the drawings as follows:

In the case of point 15, located at the center of the pole face,

line (15)–(34) — .476 centimeters
" (34)–(32) — 8.610 "
" (15)–(32) — 8.69 "
" (15)–(30) — 12.00 "

The results obtained by the two operations (*a*) and (*b*), are each taken twice in obtaining the total summation for point (15).

In the case of point 20,

line (20)–(63) — 4.33 centimetres
" (63)–(36) — 1.20 "
" (63)–(31) — 7.16 "
" (17)–(36) — .476 "
" (30)–(17) — 4.13 "
" (36)–(33) — 3.61 "
" (20)–(17)–(33) — 7.82 "
" (30)–(17)–(30) — 22.13 "

With these statements, connecting the data with the figure the calculations for these two points will be readily followed. The data for the other points (16), (17), (62), (18) and (19), see Fig. 9, was obtained in a similar way.

From the record of the "totals" in the left hand column of the second section of Table 1, the curve D of Fig. 13, has been plotted relatively to properly spaced points of the pole face and tips developed along the base line. It will be noticed that this curve is more nearly flat topped than curve C, and that after the pole corners are passed the curve drops *very* suddenly. This form of distribution over the face and corners of the pole is what we would expect, and the result is thoroughly in accord with the usual assumption in air gap calculations. It remains to be seen, however, how much of the pole tip flux finds its way into the armature.

To determine this the area (15)–(*a*)–(*b*)–(17)–(16)–(15) between the tops of curves C and D was integrated. Then the section

(24)–(c)–(d)–(24) was laid off equal in area to (15)–(a)–(b)–(17)–(16)–(15), and the area (b)–(c)–(d)–(20)–(b) integrated. Finally, when the area (17)–(62)–(18)–(u)–(20)–(b)–(17) was laid off equal in area to (b)–(c)–(d)–(20)–(b), the point (18) on curve D was located, beyond which no flux enters the armature, since the pole face flux area (15)–(17)–(18)–(u)–(16)–(15) is equal to the area of the armature surface flux area (a)–(b)–(c)–(24)–(16)–(a). The point on the pole at which the lines of force cease to enter the armature from the pole is then the point (18) of Fig. 9.

When the leakage field is estimated the armature should be removed, and it is therefore shown as a dotted line in Fig. 12, as from this figure is obtained the data for the proportional leakage field ordinates recorded in the third section of Table 1.

The summation diagrams for two points, (17) and (22), are shown; referring to the table and the figure we find that for (17)

the line (17)–(49)	— 11.15	centimeters
" (49)–(39)	— 4.60	"
" (49)–(38)	— 8.78	"
" (49)–(37)	— 25.86	"
" (17)–(43)	— 3.56	"
" (43)–(46)	— 11.55	"
" (43)–(47)	— 17.55	"
" (17)–(44)	— 7.92	"
" (44)–(47)	— 16.50	"
" (44)–(48)	— 22.50	"
" (17)–(40)	— 23.36	"
" (17)–(40)–(41)	— 27.70	"
" (17)–(40)–(42)	— 44.80	"

The proportional flux multiplier used in connection with partial summations (b) and (f), point (17), and to be applied in equations (4) and (3), has the form

$$\frac{2\,x_m - x_n - x}{x_m - x_n}.$$

In the case of point (22),

line (22)–(45)	— 26.2	centimeters
" (45)–(37)	— 10.8	"
" (45)–(38)	— 6.3	"
" (45)–(39)	— 10.4	"
" (22)–(39)	— 28.3	"
" (22)–(39)–(40)–(41)	— 44.4	"
" (22)–(39)–(40)–(42)	— 61.5	"

In the case of point (22) and the summation (45)–(37), since the proportional potential between points (22) and (45) is equal to

$$\frac{\text{line (37)–(45)}}{\text{line (37)–(38)}} = .68$$

and the potential between points (22) and (37) is zero, the constants of the correct proportional M. M. F. factor have the following values:

$$A = \alpha_m \text{ and } B = 1.588\ \alpha_m$$

Again, since the proportional difference of potential between point (22) and (38) is one, in the summation over the surface (45)–(38),

$$A = 1.703\ \alpha_m \text{ and } B = 2.703\ \alpha_m.$$

The proportional leakage field ordinates tabulated in the third section of Table 1, for the points (17,) (18), (19) and (20) are

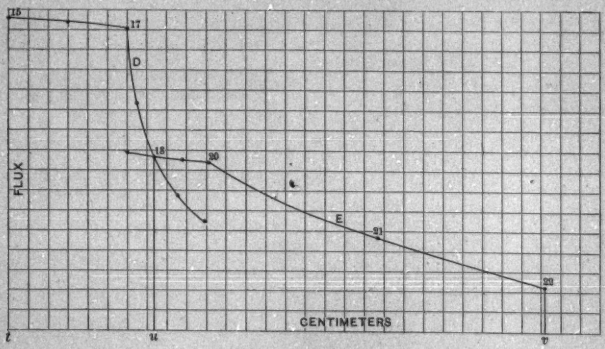

FIG. 15.

plotted over properly spaced divisions on the base line of Fig. 13. It is seen that the curve E determined in this way crosses the curve D very nearly at the point where the pole face flux ceases to flow into the armature. This checks the first calculations, as of necessity there must be the same density on either side of the point on the pole tip at which the flux divides to pass to the armature and the adjacent pole respectively.

To determine the leakage co-efficient of the machine, approximately, we can plot the results contained in the second and third sections of Table 1, and by integrating the areas they inclose obtain the necessary data. Fig. 15 shows the result of this plot.

The area of (15)–(17)–(18)–(*u*)–(*t*)–(15) is 6.7 units, and the area of (18)–(20)–(21)–(22)–(*v*)–(*u*)–(18) is 6.6 units, therefore the leakage co-efficient is approximately

$$\lambda = \frac{6.7 + 6.6}{6.7} = 1.98$$

Experimental Results.

We have now arrived at a point at which we can profitably review certain of the experimental results that have been ob-

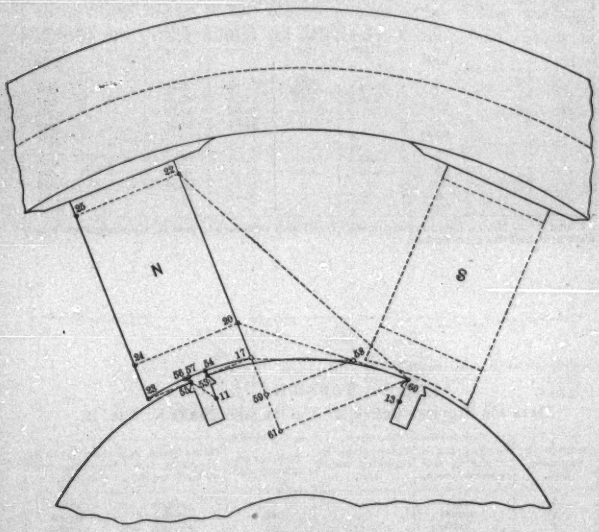

Fig. 16.

tained. The experiments were performed during the spring of the present year by Messrs. Nathan Kohn and L. S. Arnot,[1] and the records made by them bear every evidence of painstaking care. One set of readings which they obtained shows the amount of flux which enters the top of a tooth when the tooth is placed in

1. "Special Investigation of the Characteristics of the Monocyclic Generator," a Thesis presented to the Faculty of Purdue University by Leroy S. Arnot and Nathan Kohn, for the Degree of Bachelor of Science in Electrical Engineering, June, 1900.

TABLE I. SECTION 4.

Slot Flux Intensity Distribution Curve Data.

Number of figure from which data was obtained	Number of point from which summation is taken.	Data designation letter for separate integrations.	a	b	x_n	x_m	Number of formula used in integration.	Partial and total summation values.
		a	3.02	0	0	2.25	2	.089
		b	.476	2.54	0	5.10	7	1.011
		c	0	0	7.64	11.77	1	.431
		d	0	0	11.77	28.90	3	.517
16	11	e	.476	3.17	.35	5.10	7	.820
		f	0	0	8.27	12.40	1	.405
		g	0	0	12.40	29.53	3	.500
		total						4.373
		a	5.39	0	0	1.05	2	.198
		b	0	0	10.05	14.18	1	.343
	12	c	0	0	14.18	31.31	3	.430
		d	.476	5.00	.10	5.10	7	1.057
		total						4.096
		a	10.05	8.79	4.12	8.25	5	.207
		b	14.90	2.54	12.30	29.40	8	.333
16	13	c	0	0	19.64	35.80	1	.604
		d	0	0	35.80	52.90	3	.210
		total						1.354
		a	10.05	11.26	4.12	8.25	7	.100
		b	14.90	5.00	12.30	29.40	8	.302
	14	c	0	0	22.10	38.30	1	.548
		d	0	0	38.30	55.40	3	.194
		total						1.224

Note: Point No. 12, being symmetrically placed with reference to pole N, the summation is only taken over half the polar surface.

TABLE II

Data for Plotting Curve M., Fig. 14, and Curve N., Fig. 18.

Position of Exploring Coil.	Values given by exploring coil around tooth.	Values given by exploring coil in slot.	Position of exploring coil.	Values given by exploring coil around tooth.	Values given by exploring coil in slot.
6	245000		27	514900	28600
7	102600		28	969800	163500
8	17580		29	1322200	215880
9	73000		30	1416400	239088
10	107000		31	1400840	242060
11	422600		32	1372700	235560
12	769400		33	1403600	265000
13	1128600		34	1403600	276670
14	1293600		35	1217300	209032
15	1307100		36	835800	238800
16	1408400		37	441275	127000
17	1400800		38	176200	80000
18	1392100		39	63000	69000
19	1351330		40	11800	45000
20	1033800		41	113100	28830
21	588300		42	269800	4400
22	247500	56400	43	622300	58000
23	109600	40650	44	1007670	193650
24	574	21500	45	1322200	209500
25	77700	9700	46		224050
26	292800	6300	47		218800

various positions relatively to the pole. The data for this set is tabulated in the center column of Table 2, and is plotted in Fig 14, being shown as the curve M. In this figure we have then a comparison between carefully calculated and carefully explored armature surface flux distribution curves. The curves follow one another with remarkable accuracy, the variations between them at the positive and negative crests where they depart most from one another, being but between two and three per cent. even in the case of the points at plotting position 30, Fig. 14.

Regarding curve M the experimenters say: "The readings obtained are very symmetrical relatively to the poles. There is

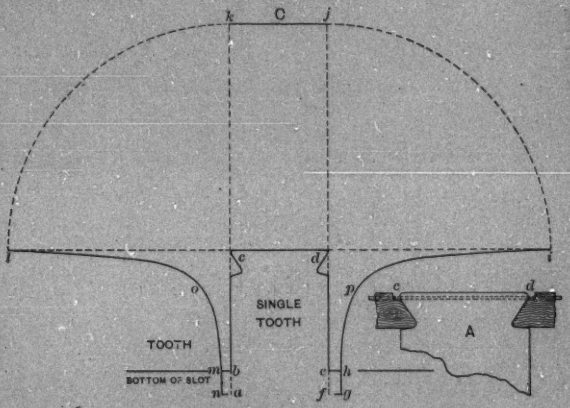

FIG. 17.

a slight tendency to draw toward the pole in the direction in which the armature is moved. The indentation at the apex of the second curve could be due to a sudden variation in the length of the air gaps, or a point of low permeance in the iron of the pole face."

In Fig. 18 is shown the plot of the data in the left hand column of Table 2. Its ordinates are proportional to the flux entering a slot as the armature is moved under the poles. "The curve has a very good symmetry. The zero points of the curve are, however, very much drawn in the direction of motion of the armature. This is due to the residual magnetism which the iron

retains while leaving the fringing field, and it is necessary to move the slots some distance into the opposite fringing field before the residual magnetism is counteracted and the polarity of the iron in the slot reversed."

Curves M, Fig. 14, and N, Fig. 18, were obtained by winding exploring coils around a tooth and in a slot as indicated, and using a ballistic galvanometer method. The current in the field circuit was reversed by a quick-acting switch several times in making the record for each reading, and the average results taken. The ordinates of the curve M represent accurately the amount of flux entering the top of a tooth, but the ordinates of the curve N are thought to be a few per cent. smaller than the actual values as some difficulty was experienced in getting the slot coil to hug

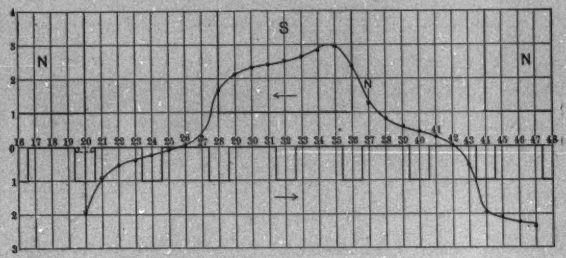

FIG. 18.

the sides of the teeth closely. As the density on the side faces of the teeth diminish very rapidly from the corner of the tooth downward, a very slight opening between the sides of the coil and the edges of the teeth would reduce the deflection given by the coil considerably below the normal.

However this may be, the results show that in the maximum positions a slot takes actually 21 per cent. as much flux as the face of a tooth, or proportionally 46 per cent. of the flux, as a tooth is 2.2 times as wide as a slot.

The experimenters also record the data for curves O and P plotted in Figs. 19 and 20 respectively. Curve O of Fig. 19 is plotted from the data tabulated in the center column of Table 3. The data was obtained by using a complete phase winding as an exploring coil. Curve P of Fig. 20, was plotted from data re-

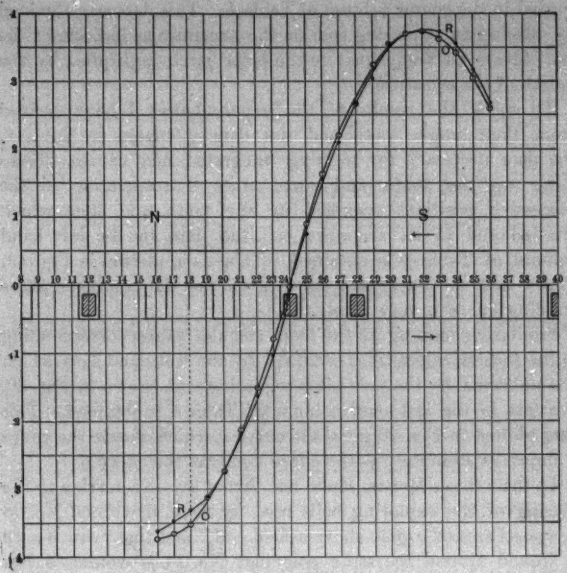

FIG. 19.

TABLE III.

DATA FOR PLOTTING CURVES IN O AND R, FIG. 19.

Position of exploring coil.	Values given by armature winding.	Values integrated from data in table 2.	Position of exploring coil.	Values given by armature winding.	Values integrated from data in table 2.
16	3744000	3645000	28	2680000	2692000
17	3672000	3466000	29	3257000	3048000
18	3524000	3328000	30	3572000	3571000
19	3120000	3111000	31	3702000	3693000
20	2740000	2773000	32	3781000	3731000
21	2118000	2314000	33	3624000	3755000
22	1506000	1629000	34	3440000	3593000
23	776000	1015000	35	3022000	3190000
24	7000	74000	36	2606000	2695000
25	918000	764000			
26	1655000	1567000			
27	2228000	2117000			

corded in the center column of Table 4. This data was obtained by placing an exploring coil on the armature as indicated; the coil covers an area equal to the polar pitch multiplied by the length of the armature.

To check the results of the experimenters I have derived from their single tooth and slot data tabulated in Table 2, the data necessary for plotting integrated flux curves. Summing up the tooth and slot values corresponding to the successive positions occupied by the teeth and slots which are inclosed in an armature coil, the data in the left hand column of Table 3 is obtained. This data, plotted in Fig. 19, shows that the integrated curve R checks the complete phase winding curve O very nicely.

Similarly summing up the tooth and slot values corresponding to the successive positions occupied by the teeth and slots which are embraced by the pole pitch exploring coil, shown in Fig. 20, the data in the left-hand column of Table 4 is obtained. This data plotted in Fig. 20 gives the curve S, which checks the curve P almost as accurately as the curve R checks the curve O.

In their thesis Messrs. Kohn and Arnot say, in referring to the placing of the exploring coils used by them, that "coil No. 18 is wound around a pole above a field coil," and that "coil No. 19 is wound around the same pole that No. 18 is wound on, but is located below the field coil." As the field coil on pole N of Fig. 10, occupies the space between the lines (22)–(25) and (20)–(24) the exploring coils were placed respectively just above and below the lines mentioned. They further say that "coil No. 4 is wound around four teeth and four slots." This is the exploring coil shown in Fig. 20, the data obtained with which is tabulated in Table 4. The experimenters record the following maximum readings:

Maximum for coil No.	4,	4,074,000	Maxwells
" " "	18,	7,677,000	"
" " "	19,	5,765,000	"

From this data it appears that between the line (22)–(25) of Fig. 10 and the armature surface 3,603,000 maxwells leak away or that the machine has a leakage factor or co efficient of $7{,}677{,}000 \div 4{,}074{,}000 = 1.88$.

This agrees reasonably well with the calculated value of 1.98, but it should not be compared with it without modification.

In obtaining the integrated curve value of 3,918,000 maxwells, recorded in Table 4, opposite position (31), the calcula-

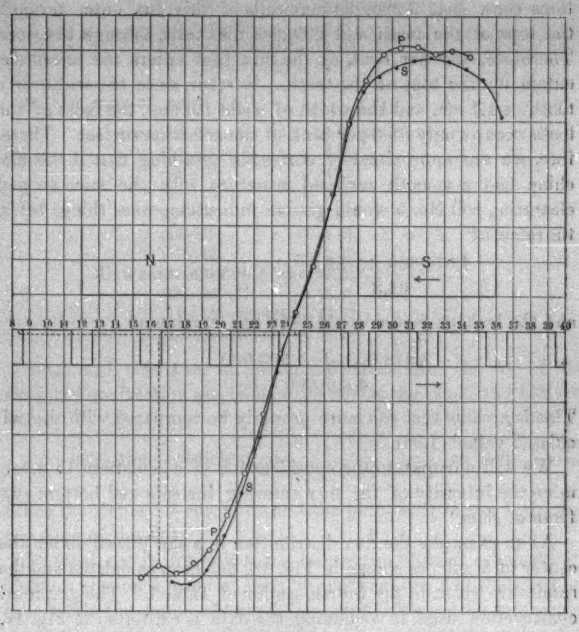

FIG. 20.

TABLE IV.

DATA FOR PLOTTING CURVES P AND S, FIG. 20.

Position of Exploring Coil.	Values given by exploring coil.	Values integrated from data in table 2.	Position of exploring coil.	Values given by exploring coil.	Values integrated from data in table 2.
16	3523000	3595000	28	2967000	3641000
17	3349000	3605000	29	3596000	3774000
18	3479000	3403000	30	3998000	3878000
19	3305000	2927000	31	4074000	3918000
20	3131000	2309000	32	4074000	3855000
21	2635000	1502000	33	3998000	3731000
22	2042000	586000	34	4023000	3600000
23	1175000	208000	35	3923000	3068000
24	313000	852000			
25	270000	1768000			
26	935000	2745000			
27	1962000	3132000			

tions show that 3,220,000 maxwells of this flux enter through the tops of the teeth and 699,000 maxwells, through the slots. Therefore, 82.2 per cent. of the flux that enters the armature, enters by the tops of the teeth. Further, since the width of a tooth is 4.1 cm. and the width of a slot 1.9 cm., the faces of the teeth occupy only 68.3 per cent. of the armature surface. Therefore, we are approximately correct in assuming that if the machine had a smooth surfaced armature with the same air gap clearance, coil No. 5 would give an indication, other things being the same, of

$$\frac{4{,}074{,}000 \times 82.2}{68.3} = 4{,}903{,}000 \text{ maxwells}$$

and the leakage factor would have had a value of

$$\frac{3{,}918{,}000 + 4{,}903{,}000}{4{,}903{,}000} = 1.80$$

This is a value that can more properly be compared with the calculated value[1] (1.98).

We will now pass to the consideration of calculations to determine the intensity of the flux entering the side and bottom surfaces of slots.

As an example, the flux distribution in a slot placed under the center of a pole, as shown in Fig. 16, has been calculated and the results are given in the fourth section of Table 1. The graphical construction used in obtaining the data is exhibited in Fig. 16, for the points (11) and (13).

In the case of point (11).

line (11)–(54)	—	8.02 centimeters
" (54)–(56)	—	2.25 "
" (53)–(54)	—	.476 "
" (11)–(53)	—	2.54 "
" (54)–(17)	—	5.10 "
" (11)–(53)–(17)	—	7.64 "
" (11)–(53)–(17)–(20)	—	11.77 "
" (11)–(53)–(17)–(22)	—	28.90 "
" (55)–(57)	—	.476 "
" (11)–(55)	—	3.17 "
" (57)–(56)	—	.35 "

1. Had the more exact method for calculating the leakage outlined in the footnote on page 81 been employed in this practical case, leakage coefficient results agreeing more exactly with the experimental values would have been obtained. This was not done, as the chief value of the method is in its application to armature surface flux distribution curve calculations.

line (57)–(23) — 5.10 centimeters
" (11)–(55)–(23) — 8.27 "
" (11)–(55)–(23)–(24) — 13.40 "
" (11)–(55)–(23)–(25) — 29.53 "

In the case of point (13),

line (58)–(59) — 10.05 centimeters
" (60)–(58)–(13) — 8.79 "
" (59)–(17) — 4.12 "
" (59)–(20) — 8.35 "
" (60)–(61) — 14.90 "
" (13)–(60) — 3.54 "
" (61)–(20) — 12.30 "
" (61)–(22) — 29.40 "
" (13)–(60)–(17) — 19.64 "
" (13)–(60)–(23)–(24) — 35.80 "
" (13)–(60)–(23)–(25) — 52.90 "

Since both points are at the same distance from the top of the slots, the contraction co-efficient for each is the same, and has the value

$$\text{contraction coefficient} = \frac{1.9}{\pi \times 2.54 + 1.9} = .192$$

Since the slot is 1.9 cm. wide and the point is 2.54 cm. from the top edge of the slot.

It must not be forgotten that before the "totals" tabulated in Table 1, section 4, can be used in plotting the flux distribution in a tooth the value obtained for point (13) must be subtracted from the value obtained for point (11), and that the value for point (14) must be subtracted from point (12). This is necessary as the values for points (11) and (12) correspond to "curve A" ordinates of the slot under the N pole of Fig. 16, while the values for points (13) and (14) correspond to similarly situated ordinates of the "curve B" of the same slot under the same pole. And, as in the case of the armature surface flux distribution curves already discussed, the ordinates of the slot "A" and "B" curves must be subtracted to get the values of the ordinates of the actual slot flux distribution. These ordinates combined with the armature surface density curve ordinates corresponding to the edges (o) and (d) of the teeth bounding the slot, give, when plotted, the distribution curves shown in Fig. 17. It will be noticed that the surface density at the sides of the slots diminishes very rapidly until a point about half way down the tooth is reached, after which it falls off more slowly. When the bottom of the slot is reached the flux density has diminished until it is only 4.2 per

cent. as great as the density at the top of the tooth. Half way down the side of the tooth the density is 7.9 per cent. as great as it is at the top.

The area (*c–k–j–d–c*) is proportional to the calculated flux entering the top of a tooth, and the area (*a–b–c–l–o–m–n–a*) plus (*f–e–d–i–h–g–f*) is proportional to the calculated flux entering the armature core through a slot. Taking the ratio of the latter to the former we find that the calculated results indicate that 37 per cent. as much flux enters the armature core through the slots as through the teeth, or that 27 per cent. of the total flux enters the armature by the slots.

It has already been pointed out that if we compare the actual maximum tooth and slot values, tabulated opposite positions (33) and (35) of Table 2, the comparison shows that 21 per cent. as much flux enters by the slots as by the teeth. If we compare the tooth and slot flux values that are tabulated opposite position (35) in Table 2, *i. e.*, if we compare the value of the flux that enters the tooth when the tooth is in the position which the slot occupies when maximum flux enters the slot, with this maximum slot flux, the comparison shows that 25 per cent. as much flux enters the armature core by the slots as by the teeth.

These ratios of experimental flux values are from a half to one-third less than the ratio of the calculated values. But this does not conclusively prove that the calculated percentage is too[1]

1. Messrs. Herdt and Archibald in a paper on "Conditions Affecting the Wave Form of Alternators," read before the Canadian Electrical Association, Aug. 30, 1900, show the assemblage of parts of a "revolving field distributed winding alternator" reproduced in Fig. B, and say in referring to the machine:

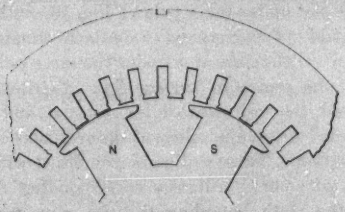

FIG. B.

"As might have been expected, the larger part of the induction enters the armature through the teeth, but about one-third, in the case of those teeth immediately opposite the pole, passes into the armature by way of the slot."

"*About one-third*," presumably, of the *total* flux.

high. The experimental results show [see Table 2, position (35)] that when a maximum of 300,000 maxwells were flowing into the slot, 1,217,000 maxwells were flowing into the tooth, and to obtain a ratio of .37 instead of .25 (the ratio these numbers give), would only require the loss of 50,000 maxwells on each side of the tooth, and a gain of a corresponding number by the coil. As it was necessary in the experiments, in order to hold the exploring coils in place, to cut the wooden slot caps as indicated at A, Fig. 17, points (*c*) and (*d*), it is quite likely that there was at least this great error in the results. The flux density changes in intensity so rapidly about the corners of the teeth that an aggregate variation of one-sixteenth of an inch in the position of the exploring coils would account for the discrepancy between the calculated and experimental results.

Conclusion.

This paper has been prepared for the Institute with a view to placing on record a crucial test of a method for calculating flux distribution. The general applications previously cited pointed to the probability of its giving indications quite comparable to actual conditions, and these latest results tend to substantiate such surmises. The method lends itself with almost equal readiness to the calculation of stray fields about armature ends, that it does to the calculation of inter-polar reactions, and by suitable modifications in the formulas, can be used for general electromagnetic calculations in which the law of "inverse squares" is even more nearly approximated than in the case cited.

Electrical Laboratory,
Purdue University,
December, 1900.

Appendix.

For the benefit of those interested in the detail of making the calculations, the full set of substitutions made in determining the complete summation of the reciprocals at the point (10) and from the pole N, are here stated. For the graphical construction see Fig. 10, and for the explanation of this construction see page 47.

Data for Partial Summations.

Summation	a	b	x_n	x_m	Formula
a	9.8	7.4	4.06	8.20	(5)
b	15.5	0	12.95	30.08	(4)
c	0	0	18.00	34.13	(1)
d	0	0	34.13	51.26	(3)

$$y_a = \log_e \frac{8.2 + \sqrt{\overline{9.8}^2 + \overline{8.2}^2}}{4.06 + \sqrt{\overline{9.8}^2 + \overline{4.06}^2}} -$$

$$\frac{14.8}{\sqrt{\overline{9.8}^2 - \overline{7.4}^2}} \left(\tan^{-1} \frac{8.2 + \sqrt{\overline{9.8}^2 + \overline{8.2}^2} + 7.4}{\sqrt{\overline{9.8}^2 - \overline{7.4}^2}} \right.$$

$$\left. \tan^{-1} \frac{4.06 + \sqrt{\overline{9.8}^2 + \overline{4.06}^2} + 7.4}{\sqrt{\overline{9.8}^2 - \overline{7.4}^2}} \right)$$

$$= \log_e \frac{20.92}{14.66} - \frac{14.8}{6.4} \left(\tan^{-1} \frac{28.32}{6.4} - \tan^{-1} \frac{22.06}{6.4} \right)$$

$$= 2.304 \log_{10} 1.43 - 2.31 \left(\tan^{-1} 4.42 - \tan^{-1} 3.45 \right)$$

$$= 2.304 \times 0.155 - 2.31 \times \frac{\pi}{180^\circ} (77.25^\circ - 73.83^\circ)$$

$$= .357 - .137 = .22$$

$$y_b = \frac{30}{17.1} \log_e \frac{30 + \sqrt{15.5^2 + 30^2}}{12.95 + \sqrt{15.5^2 + 12.95^2}} - \frac{\sqrt{15.5^2 + 30^2} - \sqrt{15.5^2 + 12.95^2}}{17.1}$$

$$= 2.304 \times 1.75 \log_{10} 1.92 - .795 = .35$$

$$y_c = \log_e \frac{34.1}{18} = 2.304 \log_{10} 1.89 = .636$$

$$y_d = \frac{51.26}{17.13} \log_e \frac{51.26}{34.13} - 1 = 2.304 \times 3 \log_{10} 1.50 - 1 = .21$$

and therefore

$y_{10} = .22 + .35 + .636 + .21 = 1.416$, which is the value of the *proportional* flux density ordinate at point (10).

Discussion.

[See p. 233 April 1900 issue.]

Mr. Steinmetz:—I am sorry to say that I cannot agree with the conclusions derived by Mr. Townsend, nor with the reasoning leading thereto. It seems to me that eddy currents in the iron of a transformer are secondary electric currents, just as any other secondary electric current, and must therefore follow the laws of electric currents; that is, the electromotive force is proportionate to the magnetic flux enclosed by the current turns, other things remaining the same. The small or large mutual induction between the secondary and primary current does not affect the law. The mutual induction between the primary and secondary may be extremely small. Take, for instance, the case of Fig. 2 on page 320, where $b\ a\ c$ represents the eddy current flowing in one of the lamina of the iron. With the thickness of the lamina, and the frequency used in commercial transformers, the magneto-motive force of these eddy currents is small compared with that of the exciting current. Its M. M. F. is not noticeably affected by that of the eddy. If the magneto-motive force should be larger than the magneto-motive force of the exciting current, the same law still holds. It is true that experiment shows that the loss of energy by eddy currents does not always follow the law of squares, but deviates therefrom, but the cause of this deviation can be found. In cases where the eddy currents are large, it is the poorly laminated iron and the change in the permeability of the iron. Another cause is the high temperature coefficient of the iron causing a rapid decrease of its conductivity, thereby decreasing the energy loss. The total loss in the apparatus when started up fresh is very much larger than after it has been warmed up. Another cause is the magnetic stray field. In a revolving apparatus with considerable eddy current losses the total loss of energy is often more in proportion to the square law. The beginning of the break down of the insulation, and the decreased permeability of the iron makes the stray field, and the stray field reaches conducting masses outside of the circuit. It may very well be that these effects combined make the eddy current losses appear to follow, as stated by the paper, the law of the 1.6th power. This however appears to me an exceptional case. As a rule the eddy current losses can be calculated from the electric conductivity and the thickness of the iron, and compared with the losses found by tests over a wide range. In wattmeter readings there is a liability to constant error, which apparently changes the law followed by the losses. In Figs. 5 and 6, it rather contradicts the law of the 1.6th power. The observed value rises more rapidly than the calculated value. Now coming to the theoretical reasoning leading up to this 1.6th power, I may remark in regard to the statement on the first page, that in 1899 it was found that the phase of the exciting current of a transformer

was practically independent of the frequency. I noticed that thing some years ago, and published it in 1897, that the exciting current-phase does not depend on the frequency, nor on the shape of the magnetic circuit, the length of the iron and cross-section. The formula is given on page 234 of my book. Now coming to the conclusion on page 235, leading up to the 1.6th power, "*Cb* should also be a complex harmonic current wave, similar in shape and following the same laws as those which govern the primary current." This is not the case and cannot be the case. The primary current has a very characteristic shape. It tends to a peak at the maximum point of induction, is strongly bulged out at the ascending side (making diagram). This bulging out is due only to hysteresis. The current *Cb* on pages 234 and 235, that is, the eddy current, does not expand energy by hysteresis, and therefore cannot have this bulging out at the ascending and bulging in at the descending. On the contrary, the effect is to decrease the magnetic flux. If it is distorted, the distortion should be in the opposite direction, a bulging in on the ascending side. The *Cb* does not show a bulging out at the ascending side, but is rather slightly bulged in; consequently it is in accordance with the reasoning I have just stated. Then it has not the same shape as hysteresis. This leads up to the conclusion that the energy loss of this current is proportionate to the energy loss of the exciting current, and therefore the 1.6th power cannot answer. I cannot see any reason why he should assume that the eddy currents follow the 1.6th power. Theoretically they follow the square of the frequency and the square of the induction, and may deviate therefrom by secondary effects, as change of temperature in the iron or change of conductivity and a number of other secondary effects.

MR. TOWNSEND:—The first part of this paper is given up to an explanation intended to account for the experimental results which are described in the latter portion of the paper. This explanation, as stated, is only offered as a suggestion. Mr. Steinmetz has said that he does not agree with it, and offers another explanation. The remarks he has made, coming as they do from so distinguished an authority on the subject, no doubt carry great weight. There are, however, certain points in connection with the experiments tending to bear out the speculations I have advanced.

In the first place, the curve of Fig. 4 is very nearly a simple harmonic, while that of Fig. 3 is plainly much distorted. These distortions seem to me, as the figure shows, to resemble those of the primary current as closely as might be expected under the circumstances. The circuit shown in Fig. 2 is not an absolutely perfect imitation of the path followed by the eddy currents in an iron sheet, and therefore it would be unfair to expect a closer agreement between theory and practice than that which appears from the curves.

The fact is also noteworthy that the current in the short-circuit shown in Fig. 3 has practically the same phase displacement as the primary current curve.

These characteristic peculiarities in the different curves were observed repeatedly, on different days and under different conditions, and there is little possibility of there being any mistake about them.

MR. STEINMETZ:—There is one very interesting fact borne out by this paper. In Figs. 3 and 4, I may say, the curve of the secondary current should be the same. The mutual induction between secondary and primary are different and the conditions are different. The conclusion that this current should be proportionate to the exciting current is not warranted, but a very interesting result is brought out. The tests agree that the result is 1.6, but theoretically we can say that there is inherent reason that they should follow the second power, but disturbing influences should make them deviate. This paper shows that owing to the phenomena where the disturbance takes place, the result is 1.6. In another case there might be a result of 1.7th power or any other power. This is a very interesting result, and a very complex phenomenon, but you cannot determine a conclusion from individual facts. If you can get a very good agreement between experiments and tests, then it is a very interesting conclusion. But going back still one step further, we know that the hysteresis loss follows the 1.6th power, but we know that is not an inherent, physical law. It must sometimes deviate from the law. It is a somewhat complex phenomenon which can be closely represented by the 1.6th power, but it is not a physical law. It shows that a very complex physical phenomenon can be represented by simple power law which is correct by approximation, over a wide range, as is shown in this paper.

PROF. GOLDSBOROUGH:—In connection with this discussion a point occurs to me which I think has been overlooked by the experimenters. I refer to discrepancies that may be observed when the fact is neglected that the distribution of flux across the core is not uniform. I will make a diagram to bring out the point that is in my mind. In a transformer core, or any other core, which, for instance, may be regarded as represented by a stamping of this shape (see Fig 9.), in view of the fact that the path followed by the flux is much shorter at E than it is at A, if we assume the average density through the section (A-E) of 2000 lines per square centimetre, we may have an actual distribution of the flux giving a density as low as 1000 lines per square centimetre at A, and as high as 3000 lines at E. These are the circumstances under which readings for hysteresis and eddy current losses in a transformer core are always obtained. Suppose now the average density is increased, what will be the distribution of the flux? The result is to make the distribution of

the flux across the core somewhat more uniform, as the permeability of the iron becomes less for the higher density values. If we have an average density of 7000 lines in the core, the distribution will be more uniform than in the first case and the density starting at A, at say 6000 lines, would probably run up to a maximum of only 8000 lines at E owing to the lower permeability of the iron at E than at A. In consequence the variation in the maximum and minimum values attained by the flux are respectively less and greater than proportional to the increase in the average flux density from 2000 to 7000 lines. Consequently, if you were to measure the eddy current losses alone in this transformer core, in view of the particular effect to which I call

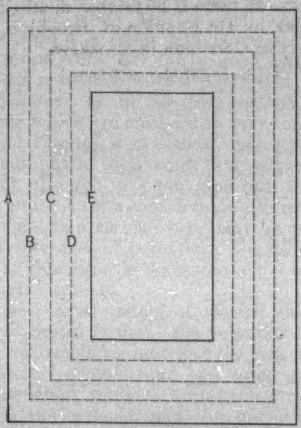

FIG. 9.

your attention, you would find that as you increase the average density in the core, the eddy current losses would increase by an amount proportional to something less than the square of the *average* flux density. It might be the 1.9th, or the 1.8th, or the 1.6th power or less, according to the kind of iron used and the variations taking place in the flux distribution. However, the actual eddy current losses in the several paths A, B, C, D and E, would all the time be strictly in accordance with the accepted rule, and proportional to the squares of the densities actually existing along the several paths.

The same thing applies to the hysteresis losses. In such a core, they increase proportionally to something less than the 1.6th power of the *average* core density, owing to the flux not being

evenly distributed over the core. The combined result is that neither the hysteresis nor the eddy current losses apparently increase as rapidly as is to be expected.

In the paper the author has neglected to take account of this matter, and has credited to the eddy current losses the entire discrepancy; assuming that the hysteresis loss varies as the 1.6th power of the *average* density under all conditions. As the hysteresis loss is always a more prominent factor than the eddy current loss, it is not surprising that, when the errors of measurement of both the major and minor losses are placed to the credit of the minor loss, the rate of increase in the minor, or eddy current loss, should appear very much less than proportional to the square of the *average* core density.

THE CHAIRMAN:—If there is no further discussion we will take up the next paper by Dr. Sheldon of Brooklyn, on the "Conditions of Electrolytic Corrosion in Brooklyn."

[See p. 337 May 1900 issue.]

Discussion.

Dr. Kennelly:—I think there is another reason why the service pipes of gas and water systems are much more seriously corroded under practical conditions, than the mains, and that is owing to the fact that the joints in the mains offer a much higher resistance electrically than the joints which are made in service pipe. This is particularly the case with gas mains and services, and I suppose it is more or less true in the case of water mains and services. In the case of gas services, for instance, you find them almost invariably made with screw-joints between the ends of the lengths which are connected together. That is to say, a main may be perhaps 30 or 40 or 100 feet from the meter, and the pipe which connects the meter with the main is a small pipe, screwed together in sections, and the screw-joints are fairly good electrically, so that there is very little resistance in that pipe. On the other hand, the mains, which may be in ten feet sections, are connected together by joints which are electrically very inferior. In order to send a current through a system of mains you have to loop the current in and out at each joint, to a very considerable extent. A current through a main pipe can therefore only be powerful in the presence of a considerable difference of potential; whereas the current in a service pipe may be powerful with a relatively small difference of potential. The difference of potential mentioned in this paper is enormous and ought not to exist in any well planned system.

In any well-constructed system of electric railroad, with well bonded track and return feeders, the potential difference can be reduced to any reasonable amount that the expense will warrant.

Mr. R. T. Lozier:—Some time ago some one advocated an auxiliary machine which would so adjust the potential as to make the water mains the cathode, and the rails the anodes, thus depositing from the rails on to the pipes instead of vice-versa. I should like to ask Dr. Sheldon if this has been a success.

Dr. Sheldon:—The legal status of the companies in Brooklyn is rather peculiar. In the first place, the men who own the electric lighting company also own the railroad, and I believe there is something in the wind as to the getting control of the gas company. The municipal water department has a bond from the electric railway company, securing the former against any damages which may occur from electrolytic corrosion. Therefore the water department will not permit any bonding of its system of pipes, nor will it permit any electrical devices to be connected with them. I would like to say in reference to Dr. Kennelly's remarks that although the main pipes have a large resistance at the joints there is no question but what a large current flows

through the pipe. In the case mentioned in the paper of 38 service pipes having been destroyed in three years, the conditions were rather peculiar. The Brooklyn Union Gas Company is formed out of various companies that have been consolidated. This corrosion of the service pipes was near the Broadway ferry where a great many cars run during the rush hours. The ferry is near the Kent Avenue power station. Down Broadway, on one side of the street, comes one gas main which then turns the corner and runs towards the power station, coming down the same street on the other side is another gas main which turns the corner and goes in the opposite direction. To this other main were connected the 38 service pipes. In entering the building of customers the service pipes passed over the first main and were corroded at the place where they passed over. The current, which was carried by the main with which they were connected, was returned to the power house by the other main, and the rapid destruction was avoided under the advice of their electrical engineer by simply connecting the service pipes to this other main rather than the first. [Adjourned to Friday, May 18th, 1900 at 10 A. M. when discussion was continued.]

MR. CHARLES J. REED:—I do not wish to take very much time, as there are a good many papers to be read this morning, but there is one point in this connection which seems to be almost always ignored, and I think it is an important one; that is the fact that it really requires no electromotive force to cause a current to enter and emerge from an iron pipe buried in the ground. An infinitesimal electromotive force will make the current pass into the iron pipe and out in preference to going through the earth. This is due to the fact, as we all know, that the ground is a conductor solely in consequence of the fact that it contains water and saline solutions, and that wherever a current passes from an electrolyte or to an electrolyte there is electro-chemical action. The current passing to an iron pipe from an electrolyte in solution, that is, containing water, will always evolve hydrogen if there is no other chemical reaction possible there which requires the absorption of less energy. If the point at which the current leaves the pipe is capable of supplying by its electro-chemical action an equal amount of energy or a greater amount, a current will pass in and out of the pipe in preference to passing through the electrolyte itself. In the case of iron, the oxidation of iron to ferrous hydroxide, evolves 68,900 calories of heat. The decomposition of water requires, of course, the same energy which is evolved in its formation, and that of liquid water is 69,000. This amount of energy must be supplied in order to effect the evolution of hydrogen and enable the current to pass in and out of the pipe. But as 408 calories must necessarily come from outside and be absorbed in the form of heat (since that is used, not electrolyti-

cally, but in expanding the evolved gas to atmospheric pressure), it leaves the amount necessary to actually decompose water 68,596. The equivalent electromotive force, required to evolve hydrogen is 1.47 volts, and that which the oxidation of the iron produces is 1.48. So that instead of an electromotive force being required to drive this current through the iron pipe, the passage of the current through the pipe produces a slight electromotive force; that is, there is some electromotive force added to the circuit and there is a tendency for the current to flow through the pipe in preference to flowing in the earth near the pipe and parallel with it. This, of course, increases the effect which Dr. Kennelly called attention to yesterday, of the current flowing in and out around an insulating joint, or a joint of even a very little resistance; but it must not be supposed that this passage of the current in and out is limited to the joint or to the neighborhood of the joint. If there is a current flowing through the earth in a direction parallel with a piece of iron pipe, or any other piece of iron, a large portion of the current will enter the iron and pass out. We might suppose that where the current passes in there would be a tendency to reduce the oxide of iron already formed there instead of evolving hydrogen. The very fact that the formation-heat of the iron oxide is greater than that of the hydrogen oxide, shows that the iron will not be reduced; and even if it were reduced, its tendency to oxidize is so great especially in the divided form, that it would not stay reduced. The corrosion would therefore go on as if there were no reduction. As a matter of fact there could be no reduction under those circumstances, unless there was a very considerable electromotive force, that is, a very powerful current. And such reduction, if it occurred, would not retard corrosion.

Mr. Ludwig Hommel:—One or two points occurred to me during the reading of this paper. In some cases we notice cables and pipes may be negative to the earth and still be positive to the rails. This must be due to the condition of the rail system, there probably being a break or bad connection in same somewhere further away from the station, and a good connection with the negative side of the dynamo from the place where that is observed. I see no reason why electrolytic action should take place in such places as that. Also another point occurred to me, bearing on the protection of underground cables. I see that Dr. Sheldon gives several paths by which the current returns to the power-house. There is one path which may be used in some instances and which I do not see mentioned here, for protecting the cables. In some telephone construction the ærial cables are spliced directly to the underground cables, and while this may not be advisable on account of the danger of burn-outs in the underground system, in case the overhead system should become

crossed with trolley wires or electric light wires, the lead covering of these ærial cables can be used as a special return to lead back the current from the underground cables to the power-house, wherever the ærial cables run toward the power-house and copper can be saved by doing so. It is a small point but very often it saves copper, and it might be interesting.

THE PRESIDENT:—If there is no further discussion we will proceed with the next paper on the list, by Dr. Perrine and Mr. Baum, on "The Use of Aluminium Line Wire and Some Constants for Transmission Line."

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